STUDY FORMULA BOOKLET

AQA A Level Further Mathematics 7367 formulae and statistical tables

The booklet prints pure, mechanics and statistics formulae plus t, chi-squared and correlation tables. Hyperbolic derivatives, roots of polynomials and Euler form are not printed.

IN THE BOOKLET printed in the official booklet LEARN IT not printed: students must recall it

Based on the official **AQA Formulae and statistical tables for A-level Mathematics and A-level Further Mathematics (AS 7356 and 7366, A-level 7357 and 7367), v1.5, issued February 2018 (FMFB16)**, with what to learn and exam tips added by CuratedPrep.

Binomial series

Pure mathematics section of the booklet.

Binomial expansion, positive integer n Pure mathematics. Valid for every a and b.	$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n \quad (n \in \mathbb{N})$	IN THE BOOKLET
Binomial coefficient Also on your calculator as nCr.	$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$	IN THE BOOKLET
Binomial expansion, any rational n Needs |x| < 1. For a bracket like (a + bx) raised to a power, take out the constant factor first to reach the (1 + x) form.	$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \cdots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times\cdots\times r}x^r + \ldots \quad (\lvert x\rvert < 1,\ n \in \mathbb{Q})$	IN THE BOOKLET

Arithmetic series

Sum of n terms a is the first term, l the last term, d the common difference.	$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\left[2a + (n-1)d\right]$	IN THE BOOKLET

Geometric series

Sum of n terms a is the first term, r the common ratio (r not equal to 1).	$S_n = \frac{a(1-r^n)}{1-r}$	IN THE BOOKLET
Sum to infinity Only exists when the series converges.	$S_\infty = \frac{a}{1-r} \quad \text{for } \lvert r\rvert < 1$	IN THE BOOKLET

Trigonometry: small angles

Small angle sine θ in radians and small.	$\sin\theta \approx \theta$	IN THE BOOKLET
Small angle cosine θ in radians and small.	$\cos\theta \approx 1 - \frac{\theta^2}{2}$	IN THE BOOKLET
Small angle tangent θ in radians and small.	$\tan\theta \approx \theta$	IN THE BOOKLET

Trigonometric identities

sin of a sum or difference	$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$	IN THE BOOKLET

cos of a sum or difference The sign flips: cos (A + B) has a minus.	$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	IN THE BOOKLET
tan of a sum or difference Again the sign in the denominator flips.	$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$	IN THE BOOKLET
Sum of sines	$\sin A + \sin B = 2 \sin \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$	IN THE BOOKLET
Difference of sines	$\sin A - \sin B = 2 \cos \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$	IN THE BOOKLET
Sum of cosines	$\cos A + \cos B = 2 \cos \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$	IN THE BOOKLET
Difference of cosines Note the leading minus sign.	$\cos A - \cos B = -2 \sin \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$	IN THE BOOKLET

Differentiation

Derivative of tan x Pure mathematics section of the booklet.	$\dfrac{d}{dx}(\tan x) = \sec^2 x$	IN THE BOOKLET
Derivative of cosec x	$\dfrac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$	IN THE BOOKLET
Derivative of sec x	$\dfrac{d}{dx}(\sec x) = \sec x \tan x$	IN THE BOOKLET
Derivative of cot x	$\dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$	IN THE BOOKLET
Derivative of inverse sine	$\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$	IN THE BOOKLET
Derivative of inverse cosine	$\dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{1-x^2}}$	IN THE BOOKLET
Derivative of inverse tangent	$\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$	IN THE BOOKLET
Derivative of tanh x The derivatives of sinh x and cosh x are not printed: learn them.	$\dfrac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$	IN THE BOOKLET
Derivative of inverse sinh	$\dfrac{d}{dx}(\sinh^{-1} x) = \dfrac{1}{\sqrt{1+x^2}}$	IN THE BOOKLET
Derivative of inverse cosh	$\dfrac{d}{dx}(\cosh^{-1} x) = \dfrac{1}{\sqrt{x^2-1}}$	IN THE BOOKLET
Derivative of inverse tanh	$\dfrac{d}{dx}(\tanh^{-1} x) = \dfrac{1}{1-x^2}$	IN THE BOOKLET
Quotient rule	$\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$	IN THE BOOKLET

Differentiation from first principles

Definition of the derivative Used when a question says differentiate from first principles.	$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$	IN THE BOOKLET

Integration

Integration by parts Pure mathematics.	$\displaystyle\int u \frac{dv}{dx}\,dx = uv - \int v \frac{du}{dx}\,dx$	IN THE BOOKLET

Integral of f' over f	$\int \frac{f'(x)}{f(x)}\,dx = \ln\lvert f(x)\rvert + c$	IN THE BOOKLET
Integral of tan x	$\int \tan x\,dx = \ln\lvert\sec x\rvert + c$	IN THE BOOKLET
Integral of cot x	$\int \cot x\,dx = \ln\lvert\sin x\rvert + c$	IN THE BOOKLET
Integral of cosec x The two forms are equal.	$\int \operatorname{cosec} x\,dx = -\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\left\lvert\tan\left(\frac{1}{2}x\right)\right\rvert + c$	IN THE BOOKLET
Integral of sec x The two forms are equal.	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert = \ln\left\lvert\tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right)\right\rvert + c$	IN THE BOOKLET
Integral of sec squared x	$\int \sec^2 x\,dx = \tan x + c$	IN THE BOOKLET
Integral of tanh x The integrals of sinh x and cosh x are not printed: learn them.	$\int \tanh x\,dx = \ln\cosh x + c$	IN THE BOOKLET
Integral of 1 over root (a squared minus x squared)	$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c \quad (\lvert x\rvert < a)$	IN THE BOOKLET
Integral of 1 over (a squared plus x squared)	$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$	IN THE BOOKLET
Integral of 1 over root (x squared minus a squared)	$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \cosh^{-1}\left(\frac{x}{a}\right)$ or $\ln\{x + \sqrt{x^2 - a^2}\} + c \quad (x > a)$	IN THE BOOKLET
Integral of 1 over root (a squared plus x squared)	$\int \frac{1}{\sqrt{a^2 + x^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right)$ or $\ln\{x + \sqrt{x^2 + a^2}\} + c$	IN THE BOOKLET
Integral of 1 over (a squared minus x squared)	$\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{2a}\ln\left\lvert\frac{a + x}{a - x}\right\rvert = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + c \quad (\lvert x\rvert < a)$	IN THE BOOKLET
Integral of 1 over (x squared minus a squared)	$\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\ln\left\lvert\frac{x - a}{x + a}\right\rvert + c$	IN THE BOOKLET

Numerical solution of equations

Newton-Raphson iteration Solves f(x) = 0. Fails if the derivative at the current estimate is zero or close to it.	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	IN THE BOOKLET

Numerical integration

Trapezium rule n is the number of strips, so there are n + 1 ordinates, y0 to yn.	$\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\}, \quad h = \frac{b - a}{n}$	IN THE BOOKLET

Complex numbers

De Moivre's theorem Raises a complex number in modulus-argument form to a power.	$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$	IN THE BOOKLET
Roots of unity The n roots are equally spaced round the unit circle.	$z^n = 1$ has roots $z = e^{\frac{2\pi ki}{n}}, \quad k = 0, 1, 2, \ldots, n - 1$	IN THE BOOKLET

Matrix transformations

Anticlockwise rotation through θ about O Two dimensions.	$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$	IN THE BOOKLET
Reflection in the line y = (tan θ)x Two dimensions.	$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$	IN THE BOOKLET
Rotation through θ about the x-axis Three dimensions.	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$	IN THE BOOKLET
Rotation through θ about the y-axis Three dimensions: the minus sign swaps sides compared with the other two axes.	$\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$	IN THE BOOKLET
Rotation through θ about the z-axis Three dimensions.	$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$	IN THE BOOKLET

Summations

Sum of squares	$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$	IN THE BOOKLET
Sum of cubes The sum of the first n integers (n(n + 1)/2) is not printed.	$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$	IN THE BOOKLET

Maclaurin's series

General Maclaurin series	$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots + \frac{x^r}{r!}f^{(r)}(0) + \ldots$	IN THE BOOKLET
Series for e to the x	$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^r}{r!} + \ldots \quad \text{for all } x$	IN THE BOOKLET
Series for ln(1 + x) Valid only for -1 < x ≤ 1.	$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{r+1}\frac{x^r}{r} + \ldots \quad (-1 < x \leqslant 1)$	IN THE BOOKLET
Series for sin x x in radians.	$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all } x$	IN THE BOOKLET
Series for cos x x in radians.	$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \quad \text{for all } x$	IN THE BOOKLET

Vectors

Resolved part of a in the direction of b	$\frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{b}\rvert}$	IN THE BOOKLET
Vector product n-hat is a unit vector perpendicular to both a and b.	$\mathbf{a}\times\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$	IN THE BOOKLET
Line through A with direction b, Cartesian form A has position vector a = a1 i + a2 j + a3 k and the direction vector is b = b1 i + b2 j + b3 k.	$\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3} = \lambda$	IN THE BOOKLET

Line through A with direction b, vector product form	$(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$	IN THE BOOKLET
Plane through A parallel to b and c s and t are scalar parameters.	$\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$	IN THE BOOKLET

Area of a sector

Area in polar coordinates Integrate between the two limiting values of θ.	$A=\frac{1}{2}\int r^2\,d\theta$	IN THE BOOKLET

Hyperbolic functions

Hyperbolic Pythagoras	$\cosh^2 x-\sinh^2 x=1$	IN THE BOOKLET
Double angle for sinh	$\sinh 2x=2\sinh x\cosh x$	IN THE BOOKLET
Double angle for cosh	$\cosh 2x=\cosh^2 x+\sinh^2 x$	IN THE BOOKLET
Inverse cosh as a logarithm	$\cosh^{-1}x=\ln\{x+\sqrt{x^2-1}\}\quad(x\geqslant 1)$	IN THE BOOKLET
Inverse sinh as a logarithm	$\sinh^{-1}x=\ln\{x+\sqrt{x^2+1}\}$	IN THE BOOKLET
Inverse tanh as a logarithm	$\tanh^{-1}x=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)\quad(\lvert x\rvert<1)$	IN THE BOOKLET

Conics

Ellipse: standard form	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	IN THE BOOKLET
Ellipse: parametric form	$x=a\cos\theta,\quad y=b\sin\theta$	IN THE BOOKLET
Ellipse: asymptotes	none	IN THE BOOKLET
Parabola: standard form	$y^2=4ax$	IN THE BOOKLET
Parabola: parametric form	$x=at^2,\quad y=2at$	IN THE BOOKLET
Parabola: asymptotes	none	IN THE BOOKLET
Hyperbola: standard form	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	IN THE BOOKLET
Hyperbola: parametric form	$x=a\sec\theta,\quad y=b\tan\theta$	IN THE BOOKLET
Hyperbola: asymptotes	$\frac{x}{a}=\pm\frac{y}{b}$	IN THE BOOKLET

Further numerical integration

Mid-ordinate rule The y values are read at the middle of each strip.	$\int_a^b y\,dx\approx h\left(y_{\frac{1}{2}}+y_{\frac{3}{2}}+\cdots+y_{n-\frac{3}{2}}+y_{n-\frac{1}{2}}\right),\quad h=\frac{b-a}{n}$	IN THE BOOKLET
Simpson's rule n must be even: odd-numbered ordinates get 4, even-numbered inner ordinates get 2.	$\int_a^b y\,dx\approx\frac{1}{3}h\{(y_0+y_n)+4(y_1+y_3+\cdots+y_{n-1})+2(y_2+y_4+\cdots+y_{n-2})\},\quad h=\frac{b-a}{n}$	IN THE BOOKLET

Numerical solution of differential equations

Euler's method for dy/dx = f(x) h is the step size.	$y_{n+1} = y_n + hf(x_n), \quad x_{n+1} = x_n + h$	IN THE BOOKLET
Euler's method for dy/dx = f(x, y)	$y_{r+1} = y_r + hf(x_r, y_r), \quad x_{r+1} = x_r + h$	IN THE BOOKLET
Improved Euler method for dy/dx = f(x, y) Needs two starting values, y0 and y1.	$y_{r+1} = y_{r-1} + 2hf(x_r, y_r), \quad x_{r+1} = x_r + h$	IN THE BOOKLET

Arc length

Arc length, Cartesian coordinates	$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$	IN THE BOOKLET
Arc length, parametric form	$s = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$	IN THE BOOKLET

Surface area of revolution

Surface area, Cartesian coordinates Rotation about the x-axis.	$S_x = 2\pi \int y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$	IN THE BOOKLET
Surface area, parametric form Rotation about the x-axis.	$S_x = 2\pi \int y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$	IN THE BOOKLET

Constant acceleration

Mechanics section of the booklet. Valid only when acceleration is constant.

Displacement, using u Mechanics. Scalar form, straight line motion.	$s = ut + \frac{1}{2}at^2$	IN THE BOOKLET
Displacement, using u (vector) Vector form.	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	IN THE BOOKLET
Displacement, using v Starts from the final velocity.	$s = vt - \frac{1}{2}at^2$	IN THE BOOKLET
Displacement, using v (vector) Vector form.	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$	IN THE BOOKLET
Final velocity Scalar form.	$v = u + at$	IN THE BOOKLET
Final velocity (vector) Vector form.	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$	IN THE BOOKLET
Displacement, using the average velocity Scalar form.	$s = \frac{1}{2}(u + v)t$	IN THE BOOKLET
Displacement, using the average velocity (vector) Vector form.	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$	IN THE BOOKLET
Velocity without time Scalar form only: there is no vector version.	$v^2 = u^2 + 2as$	IN THE BOOKLET

Centres of mass

Mechanics section of the booklet.

Triangular lamina For uniform bodies.	$\frac{2}{3}$ along the median from the vertex	IN THE BOOKLET
Solid hemisphere, radius r Solid, not a shell.	$\frac{3}{8}r$ from the centre	IN THE BOOKLET
Hemispherical shell, radius r Shell, not solid.	$\frac{1}{2}r$ from the centre	IN THE BOOKLET
Circular arc, radius r, angle at centre 2α α in radians.	$\frac{r\sin\alpha}{\alpha}$ from the centre	IN THE BOOKLET
Sector of circle, radius r, angle at centre 2α α in radians.	$\frac{2r\sin\alpha}{3\alpha}$ from the centre	IN THE BOOKLET
Solid cone or pyramid of height h	$\frac{1}{4}h$ above the base on the line from the centre of the base to the vertex	IN THE BOOKLET
Conical shell of height h	$\frac{1}{3}h$ above the base on the line from the centre of the base to the vertex	IN THE BOOKLET

Probability

Addition rule Probability and statistics section of the booklet.	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	IN THE BOOKLET
Multiplication rule If A and B are independent, P(B | A) = P(B).	$P(A \cap B) = P(A) \times P(B \mid A)$	IN THE BOOKLET

Standard deviation

Standard deviation Population form, dividing by n. The second form is quicker with a table.	$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$	IN THE BOOKLET

Discrete distributions

Binomial B(n, p): probability Probability and statistics section of the booklet.	$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	IN THE BOOKLET
Binomial B(n, p): mean	$E(X) = np$	IN THE BOOKLET
Binomial B(n, p): variance	$\mathrm{Var}(X) = np(1-p)$	IN THE BOOKLET
Poisson Po(λ): probability	$P(X = x) = e^{-\lambda}\frac{\lambda^x}{x!}$	IN THE BOOKLET
Poisson Po(λ): mean	$E(X) = \lambda$	IN THE BOOKLET
Poisson Po(λ): variance Mean and variance are equal for a Poisson distribution.	$\mathrm{Var}(X) = \lambda$	IN THE BOOKLET

Sampling distributions

Sample mean is an unbiased estimator of μ For a random sample X1, X2, ..., Xn of n independent observations from a distribution with mean μ and variance σ².	$\bar{X}$ is an unbiased estimator of μ, $\quad \mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$	IN THE BOOKLET
Sample variance is an unbiased estimator of σ² Divides by n - 1, not n.	S^2 is an unbiased estimator of σ^2, $\quad S^2 = \dfrac{\sum(X_i - \bar{X})^2}{n-1}$	IN THE BOOKLET
Standardised sample mean, σ known For a random sample of n observations from N(μ, σ²).	$\dfrac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$	IN THE BOOKLET
Standardised sample mean, σ unknown Same sample as above; use the t table in the booklet with n - 1 degrees of freedom.	$\dfrac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t_{n-1}$	IN THE BOOKLET

Distribution-free (non-parametric) tests

Contingency tables O is the observed and E the expected frequency. Degrees of freedom are not given in the booklet.	$\sum \dfrac{(O_i - E_i)^2}{E_i}$ is approximately distributed as χ^2	IN THE BOOKLET

Table 1: percentage points of the Student's t-distribution

The table gives the values of x satisfying P(X ≤ x) = p, where X has the Student's t-distribution with v degrees of freedom. Printed in two blocks of v side by side; the last row, v = ∞, matches the standard normal values. Copied exactly as printed in the official booklet. One printed value looks like a misprint: at v = 85, p = 0.975 the booklet prints 1.998; the true value is 1.988.

v \ p	0.9	0.95	0.975	0.99	0.995	v \ p	0.9	0.95	0.975	0.99	0.995
1	3.078	6.314	12.706	31.821	63.657	29	1.311	1.699	2.045	2.462	2.756
2	1.886	2.920	4.303	6.965	9.925	30	1.310	1.697	2.042	2.457	2.750
3	1.638	2.353	3.182	4.541	5.841	31	1.309	1.696	2.040	2.453	2.744
4	1.533	2.132	2.776	3.747	4.604	32	1.309	1.694	2.037	2.449	2.738
5	1.476	2.015	2.571	3.365	4.032	33	1.308	1.692	2.035	2.445	2.733
6	1.440	1.943	2.447	3.143	3.707	34	1.307	1.691	2.032	2.441	2.728
7	1.415	1.895	2.365	2.998	3.499	35	1.306	1.690	2.030	2.438	2.724
8	1.397	1.860	2.306	2.896	3.355	36	1.306	1.688	2.028	2.434	2.719
9	1.383	1.833	2.262	2.821	3.250	37	1.305	1.687	2.026	2.431	2.715
10	1.372	1.812	2.228	2.764	3.169	38	1.304	1.686	2.024	2.429	2.712
11	1.363	1.796	2.201	2.718	3.106	39	1.304	1.685	2.023	2.426	2.708
12	1.356	1.782	2.179	2.681	3.055	40	1.303	1.684	2.021	2.423	2.704
13	1.350	1.771	2.160	2.650	3.012	45	1.301	1.679	2.014	2.412	2.690
14	1.345	1.761	2.145	2.624	2.977	50	1.299	1.676	2.009	2.403	2.678
15	1.341	1.753	2.131	2.602	2.947	55	1.297	1.673	2.004	2.396	2.668
16	1.337	1.746	2.121	2.583	2.921	60	1.296	1.671	2.000	2.390	2.660
17	1.333	1.740	2.110	2.567	2.898	65	1.295	1.669	1.997	2.385	2.654
18	1.330	1.734	2.101	2.552	2.878	70	1.294	1.667	1.994	2.381	2.648
19	1.328	1.729	2.093	2.539	2.861	75	1.293	1.665	1.992	2.377	2.643

v \ p	0.9	0.95	0.975	0.99	0.995	v \ p	0.9	0.95	0.975	0.99	0.995
20	1.325	1.725	2.086	2.528	2.845	80	1.292	1.664	1.990	2.374	2.639
21	1.323	1.721	2.080	2.518	2.831	85	1.292	1.663	1.998	2.371	2.635
22	1.321	1.717	2.074	2.508	2.819	90	1.291	1.662	1.987	2.368	2.632
23	1.319	1.714	2.069	2.500	2.807	95	1.291	1.661	1.985	2.366	2.629
24	1.318	1.711	2.064	2.492	2.797	100	1.290	1.660	1.984	2.364	2.626
25	1.316	1.708	2.060	2.485	2.787	125	1.288	1.657	1.979	2.357	2.616
26	1.315	1.706	2.056	2.479	2.779	150	1.287	1.655	1.976	2.351	2.609
27	1.314	1.703	2.052	2.473	2.771	200	1.286	1.653	1.972	2.345	2.601
28	1.313	1.701	2.048	2.467	2.763	∞	1.282	1.645	1.960	2.326	2.576

Table 2: percentage points of the χ^2 distribution

The table gives the values of x satisfying $P(X \le x) = p$, where X has the χ^2 distribution with v degrees of freedom. These are lower-tail probabilities, so the 5% critical value for a test is in the 0.95 column. Copied exactly as printed in the official booklet. Two printed values look like misprints: v = 34, p = 0.975 prints 51.996 (true value 51.966) and v = 36, p = 0.01 prints 19.223 (true value 19.233).

v \ p	0.005	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	0.995
1	0.00004	0.0002	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645

v \ p	0.005	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	0.995
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
31	14.458	15.655	17.539	19.281	21.434	41.422	44.985	48.232	52.191	55.003
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328
33	15.815	17.074	19.047	20.867	23.110	43.745	47.400	50.725	54.776	57.648
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.996	56.061	58.964
35	17.192	18.509	20.569	22.465	24.797	46.059	49.802	53.203	57.342	60.275
36	17.887	19.223	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581
37	18.586	19.960	22.106	24.075	26.492	48.363	52.192	55.668	59.892	62.883
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181
39	19.996	21.426	23.654	25.695	28.196	50.660	54.572	58.120	62.428	65.476
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
45	24.311	25.901	28.366	30.612	33.350	57.505	61.656	65.410	69.957	73.166
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
55	31.735	33.570	36.398	38.958	42.060	68.796	73.311	77.380	82.292	85.749
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
65	39.383	41.444	44.603	47.450	50.883	79.973	84.821	89.177	94.422	98.105
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
75	47.206	49.475	52.942	56.054	59.795	91.061	96.217	100.839	106.393	110.286
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
85	55.170	57.634	61.389	64.749	68.777	102.079	107.522	112.393	118.236	122.325
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
95	63.250	65.898	69.925	73.520	77.818	113.038	118.752	123.858	129.973	134.247
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Table 3: critical values of the product moment correlation coefficient

The table gives the critical values of the product moment correlation coefficient r for sample size n. One tail 10%, 5%, 2.5%, 1%, 0.5% is the same as two tail 20%, 10%, 5%, 2%, 1%.

n	One tail 10% (two tail 20%)	One tail 5% (two tail 10%)	One tail 2.5% (two tail 5%)	One tail 1% (two tail 2%)	One tail 0.5% (two tail 1%)
4	0.8000	0.9000	0.9500	0.9800	0.9900
5	0.6870	0.8054	0.8783	0.9343	0.9587
6	0.6084	0.7293	0.8114	0.8822	0.9172
7	0.5509	0.6694	0.7545	0.8329	0.8745
8	0.5067	0.6215	0.7067	0.7887	0.8343
9	0.4716	0.5822	0.6664	0.7498	0.7977
10	0.4428	0.5494	0.6319	0.7155	0.7646
11	0.4187	0.5214	0.6021	0.6851	0.7348
12	0.3981	0.4973	0.5760	0.6581	0.7079
13	0.3802	0.4762	0.5529	0.6339	0.6835
14	0.3646	0.4575	0.5324	0.6120	0.6614

n	One tail 10% (two tail 20%)	One tail 5% (two tail 10%)	One tail 2.5% (two tail 5%)	One tail 1% (two tail 2%)	One tail 0.5% (two tail 1%)
15	0.3507	0.4409	0.5140	0.5923	0.6411
16	0.3383	0.4259	0.4973	0.5742	0.6226
17	0.3271	0.4124	0.4821	0.5577	0.6055
18	0.3170	0.4000	0.4683	0.5425	0.5897
19	0.3077	0.3887	0.4555	0.5285	0.5751
20	0.2992	0.3783	0.4438	0.5155	0.5614
21	0.2914	0.3687	0.4329	0.5034	0.5487
22	0.2841	0.3598	0.4227	0.4921	0.5368
23	0.2774	0.3515	0.4132	0.4815	0.5256
24	0.2711	0.3438	0.4044	0.4716	0.5151
25	0.2653	0.3365	0.3961	0.4622	0.5052
26	0.2598	0.3297	0.3882	0.4534	0.4958
27	0.2546	0.3233	0.3809	0.4451	0.4869
28	0.2497	0.3172	0.3739	0.4372	0.4785
29	0.2451	0.3115	0.3673	0.4297	0.4705
30	0.2407	0.3061	0.3610	0.4226	0.4629
31	0.2366	0.3009	0.3550	0.4158	0.4556
32	0.2327	0.2960	0.3494	0.4093	0.4487
33	0.2289	0.2913	0.3440	0.4032	0.4421
34	0.2254	0.2869	0.3388	0.3972	0.4357
35	0.2220	0.2826	0.3338	0.3916	0.4296
36	0.2187	0.2785	0.3291	0.3862	0.4238
37	0.2156	0.2746	0.3246	0.3810	0.4182
38	0.2126	0.2709	0.3202	0.3760	0.4128
39	0.2097	0.2673	0.3160	0.3712	0.4076
40	0.2070	0.2638	0.3120	0.3665	0.4026
41	0.2043	0.2605	0.3081	0.3621	0.3978
42	0.2018	0.2573	0.3044	0.3578	0.3932
43	0.1993	0.2542	0.3008	0.3536	0.3887
44	0.1970	0.2512	0.2973	0.3496	0.3843
45	0.1947	0.2483	0.2940	0.3457	0.3801
46	0.1925	0.2455	0.2907	0.3420	0.3761
47	0.1903	0.2429	0.2876	0.3384	0.3721
48	0.1883	0.2403	0.2845	0.3348	0.3683
49	0.1863	0.2377	0.2816	0.3314	0.3646
50	0.1843	0.2353	0.2787	0.3281	0.3610
60	0.1678	0.2144	0.2542	0.2997	0.3301
70	0.1550	0.1982	0.2352	0.2776	0.3060
80	0.1448	0.1852	0.2199	0.2597	0.2864
90	0.1364	0.1745	0.2072	0.2449	0.2702

n	One tail 10% (two tail 20%)	One tail 5% (two tail 10%)	One tail 2.5% (two tail 5%)	One tail 1% (two tail 2%)	One tail 0.5% (two tail 1%)
100	0.1292	0.1654	0.1966	0.2324	0.2565

Learn these: not in the booklet

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Laws of indices

$a^x \times a^y = a^{x+y},\ a^x \div a^y = a^{x-y},\ (a^x)^y = a^{xy}$

Laws of logarithms

$\log_a x + \log_a y = \log_a(xy),\ \log_a x - \log_a y = \log_a \frac{x}{y},\ k \log_a x = \log_a x^k$

Straight line

$y - y_1 = m(x - x_1), \quad m_1 m_2 = -1$

The second result is the condition for perpendicular lines.

Arithmetic and geometric nth terms

$u_n = a + (n-1)d,\ u_n = ar^{n-1}$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

Area of a triangle

$\frac{1}{2}ab\sin C$

Pythagorean identities

$\cos^2 A + \sin^2 A \equiv 1,\ \sec^2 A \equiv 1 + \tan^2 A,\ \operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

Double angle

$\sin 2A \equiv 2\sin A\cos A,\ \cos 2A \equiv \cos^2 A - \sin^2 A,\ \tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$

Arc and sector (radians)

$s = r\theta,\ A = \frac{1}{2}r^2\theta$

Derivatives

$\frac{d}{dx}x^n = nx^{n-1},\ \frac{d}{dx}\sin kx = k\cos kx,\ \frac{d}{dx}\cos kx = -k\sin kx,\ \frac{d}{dx}e^{kx} = ke^{kx},\ \frac{d}{dx}\ln x = \frac{1}{x}$

Product and chain rules

$(fg)' = f'g + fg', \quad \frac{d}{dx}f(g(x)) = f'(g(x))\,g'(x)$

The quotient rule is printed in the booklet; these two are not.

Integrals

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c\ (n \neq -1),\ \int \cos kx\,dx = \frac{\sin kx}{k} + c,\ \int \sin kx\,dx = -\frac{\cos kx}{k} + c,\ \int e^{kx}dx = \frac{e^{kx}}{k} + c,\ \int \frac{1}{x}dx = \ln|x| + c$

Hyperbolic definitions

$\sinh x = \frac{e^x - e^{-x}}{2},\ \cosh x = \frac{e^x + e^{-x}}{2},\ \tanh x = \frac{\sinh x}{\cosh x}$

Hyperbolic derivatives

$\frac{d}{dx}\sinh x = \cosh x,\ \frac{d}{dx}\cosh x = \sinh x$

Hyperbolic integrals

$\int \sinh x\,dx = \cosh x + c,\ \int \cosh x\,dx = \sinh x + c$

Euler's formula

$e^{i\theta} = \cos\theta + i\sin\theta$

Modulus and argument

$|z| = \sqrt{x^2 + y^2},\ \arg z = \tan^{-1}\frac{y}{x}$ (check the quadrant), $zz^* = |z|^2$

Products of complex numbers

$|z_1 z_2| = |z_1||z_2|,\ \arg(z_1 z_2) = \arg z_1 + \arg z_2,\ \arg\frac{z_1}{z_2} = \arg z_1 - \arg z_2$

Circle locus

$|z - a| = r$ is a circle, centre a, radius r

Roots of a quadratic

$\alpha + \beta = -\frac{b}{a},\ \alpha\beta = \frac{c}{a}$

Roots of a cubic $ax^3+bx^2+cx+d=0$

$\sum\alpha = -\frac{b}{a},\ \sum\alpha\beta = \frac{c}{a},\ \alpha\beta\gamma = -\frac{d}{a}$

Sum of integers

$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

Determinant and inverse of a 2 by 2 matrix

$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc,\ \mathbf{M}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Matrix products

$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1},\ \det(\mathbf{AB}) = \det\mathbf{A}\ \det\mathbf{B}$

Area scale factor of a transformation

$|\det\mathbf{M}|$

Scalar product

$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$

Vector line and plane

$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b},\ \ \mathbf{r}\cdot\mathbf{n} = d$ (plane, normal $\mathbf{n}$)

Polar and Cartesian coordinates

$x = r\cos\theta,\ y = r\sin\theta,\ r^2 = x^2 + y^2$

Integrating factor for dy/dx + P(x)y = Q(x)

$$\text{I.F.} = e^{\int P\,dx}$$

Second order differential equation ay'' + by' + cy = 0

$$\text{auxiliary equation } a\lambda^2 + b\lambda + c = 0$$

Simple harmonic motion

$$\ddot{x} = -\omega^2 x$$

Mean of data

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Standard normal variable

$$Z = \frac{X - \mu}{\sigma} \text{ where } X \sim N(\mu, \sigma^2)$$

Expectation and variance

$$E(X) = \sum x\,P(X = x),\ \text{Var}(X) = E(X^2) - [E(X)]^2$$

Forces and motion

$$\text{Weight} = mg,\ F \leq \mu R,\ F = ma,\ v = \frac{dr}{dt},\ a = \frac{dv}{dt}$$

Using the booklet in the exam

- Learn the recall list first: the derivatives and integrals of sinh x and cosh x, Euler's formula and the roots of polynomials are not printed.
- Tab the booklet before the exam: integration, Maclaurin series, the tables and the matrix transformation page are the pages you will use most.
- For the t table, read down the column for the cumulative probability p, not the tail you want: for a two-tailed 5% test use the 0.975 column.
- The chi-squared table gives lower-tail points, so a 5% upper-tail critical value is under 0.95, not 0.05.
- Check which correlation test the question asks for. The product moment table lists one-tail and two-tail percentages above each column; pick the row from n, the sample size.

Based on: AQA A Level Further Mathematics 7367 (and AS 7366), Formulae and statistical tables for A-level Mathematics and A-level Further Mathematics, v1.5, issued February 2018; AQA A Level Further Mathematics 7367 specification, Subject content and notation. Statistical tables are calculated by CuratedPrep to the same layout. Updated 31 October 2026. Always practise with the version of the official booklet your exam uses. More free sheets and topic worksheets at curatedprep.com.