STUDY FORMULA BOOKLET

Cambridge 9709 Maths Paper 3: pure mathematics 3

MF19 gives the binomial series, addition and double angle formulae, derivatives, integration by parts and the scalar product. Learn first the R-formula, partial fraction forms, the separable equation method, the complex number rules and the vector line equation.

GIVEN printed in the formula list (MF19) LEARN IT not in MF19: learn it

Built by CuratedPrep from 41 past question papers for Paper 3 in our question bank, March 2020 to November 2025. Given means it is in the List of formulae and statistical tables (MF19) given in every paper. Learn it means a past question needed it and the exam did not print it.

Modulus, polynomials and partial fractions

MF19 prints none of these, so all of them are Learn it.

Modulus equations and inequalities

Sketch the V-shaped graph first, then find the critical value where the lines cross. Used in: Nov 2025 paper 32 Q1(b), an inequality with y = |x + a| (the dump has lost the modulus bars); Jun 2025 paper 31 Q1(a), a sketch of a modulus graph; Jun 2025 paper 31 Q1(b).

$$|a| = |b| \iff a^2 = b^2, \qquad |x - a| < b \iff a - b < x < a + b$$

LEARN IT

Factor and remainder theorems

For a factor (ax + b), substitute x = -b/a. Substitute twice to get two equations when two constants are unknown. Used in: Jun 2025 paper 31 Q5, two remainders giving p and q; Nov 2025 paper 32 Q5(b), (3x - 2) a factor and p and q to find; Nov 2025 paper 32 Q5(a), showing (x - a) is a factor of f'(x).

$$\mathrm{f}(a) = 0 \iff (x - a) \text{ is a factor;} \qquad \text{remainder on dividing by } (x - a) = \mathrm{f}(a)$$

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Partial fractions: forms to recall

Add a constant A when the numerator has the same degree as the denominator. Find the constants by substituting values of x or by comparing coefficients. Used in: Nov 2025 paper 31 Q10(a), a linear factor and a quadratic factor; Nov 2025 paper 32 Q9(a), a constant plus two linear factors; Jun 2025 paper 33 Q7(abc), two linear factors.

$$\frac{A}{ax + b} + \frac{B}{cx + d}, \qquad \frac{A}{ax + b} + \frac{Bx + C}{cx^2 + d}$$

LEARN IT

Binomial series

Binomial series: rational n

Rewrite (a + bx)^n as a^n(1 + bx/a)^n first, and state the validity as |bx/a| < 1. Used in: Jun 2025 paper 32 Q2(a), expand to x^2 with simplified coefficients; Jun 2025 paper 32 Q2(b), the values of x for which the expansion is valid; Jun 2024 paper 31 Q1.

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \quad (|x| < 1)$$

GIVEN

Logarithms and exponentials

Laws of logarithms

Combine logarithms, then remove them with e^(ln x) = x or by comparing arguments. Used in: Nov 2025 paper 31 Q2(a), a logarithmic equation reduced to a quadratic; Jun 2025 paper 31 Q2, q in terms of p; Jun 2024 paper 31 Q2.

$$\ln a + \ln b = \ln(ab), \quad \ln a - \ln b = \ln \frac{a}{b}, \quad \ln a^k = k \ln a$$

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Logarithms and indices

Take ln of both sides when x is in a power, then use the power law. Used in: Nov 2025 paper 32 Q2, an equation with x in the indices; Jun 2024 paper 31 Q2, removing logarithms to a quadratic.

$$a^x = b \iff x = \frac{\ln b}{\ln a}, \qquad e^{\ln x} = x, \qquad \ln e^x = x$$

LEARN IT

Trigonometry

MF19 prints the identities for $\tan^2$ and $\sec^2$, the addition formulae and the double angle formulae, but not the R-formula or the definitions of sec, cosec and cot.

Reciprocal trigonometric functions

Rewrite in sin, cos and tan to sketch the graph or to rearrange an equation into an iterative formula. Used in: Nov 2025 paper 31 Q9(bcd), sec x rewritten as a cosine in an iteration; Nov 2025 paper 32 Q6(a), a graph of cot 2x; Jun 2024 paper 31 Q6(a), a graph of cosec x.

$$\sec\theta = \frac{1}{\cos\theta}, \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta}, \qquad \cot\theta = \frac{1}{\tan\theta}$$

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Tangent and secant identity

Use it to differentiate an inverse tangent: tan y = 2x gives $\sec^2$ y dy/dx = 2, and $\sec^2$ y = 1 + $\tan^2$ y. Used in: Jun 2024 paper 31 Q10(a), showing that dy/dx = 2/(1 + $4x^2$).

$$1 + \tan^2\theta \equiv \sec^2\theta$$

GIVEN

Sine of a sum or difference

Expand, then replace sin and cos of the standard angle by their exact values. Used in: Nov 2025 paper 35 Q8(a), sin(θ + pi/3) and a related term in the form R sin(θ + alpha); Jun 2025 paper 31 Q7(a), expanding sin(x - pi/6).

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

GIVEN

Cosine of a sum or difference

Expand, then collect the sin x and cos x terms before you find R and alpha. Used in: Nov 2025 paper 31 Q3(a), expanding cos(x + 45 degrees) in an expression to write as R cos(x - alpha).

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

GIVEN

Double angle: sine

Write sin 2x as 2 sin x cos x, and sin 4x as 2 sin 2x cos 2x. Used in: Nov 2025 paper 32 Q8(a), proving an identity for sin 4x; Jun 2025 paper 31 Q11(b), a form that can be integrated; Jun 2024 paper 31 Q8, a substitution for the integral of sin x sin 2x.

$$\sin 2A \equiv 2 \sin A \cos A$$

GIVEN

Double angle: cosine

Choose the form that leaves one trigonometric function. Used in: Nov 2025 paper 32 Q8(a), the double angle for cos 2x; Jun 2025 paper 31 Q11(a), an equation in one trigonometric function for the maximum point.

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

GIVEN

R-formula

Expand the required form, compare coefficients to get R cos alpha and R sin alpha, then use the new form to solve the equation in the interval. Part (b) of the first two then solves an equation with the new form. Used in: Nov 2025 paper 31 Q3(a), R cos(x - alpha); Jun 2025 paper 31 Q7(a), R sin(x - alpha); Nov 2025 paper 35 Q8(a).

$$a\sin\theta + b\cos\theta = R\sin(\theta+\alpha); \quad a\cos\theta + b\sin\theta = R\cos(\theta-\alpha); \quad R = \sqrt{a^2+b^2}, \ \tan\alpha = \frac{b}{a} \text{ for } R\cos\alpha = a, \ R\sin\alpha = b$$

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Solutions of a trigonometric equation in an interval

alpha is the calculator value. In radians use pi and 2 pi. After an R-formula, solve for (θ + alpha) first, then subtract alpha and keep only values inside the interval. Used in: Nov 2025 paper 31 Q3(b), two solutions between 0 and 360 degrees; Jun 2025 paper 31 Q7(b), two solutions between 0 and 2 pi.

$$\sin\theta = k: \ \alpha, \ 180^\circ - \alpha; \quad \cos\theta = k: \ \alpha, \ 360^\circ - \alpha; \quad \tan\theta = k: \ \alpha, \ \alpha + 180^\circ$$

LEARN IT

Differentiation

MF19 prints the derivatives of ln x, e^x, the trigonometric functions and $\tan^{-1} x$, and the product, quotient and parametric rules. The chain rule and implicit differentiation are not printed.

Derivative of ln x

With the chain rule, d/dx ln(f(x)) = f'(x)/f(x). Used in: Nov 2025 paper 31 Q7, the y part of a parametric curve; Nov 2025 paper 35 Q4, a curve with logarithms in its equation.

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

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Derivative of e to the power x

With the chain rule, d/dx e^(f(x)) = f'(x) e^(f(x)). Used in: Nov 2025 paper 31 Q4, a maximum point of an exponential times a trigonometric function; Jun 2024 paper 31 Q5, stationary points of a quotient with e^(sin x).

$$\frac{d}{dx}\left(e^x\right) = e^x$$

GIVEN

Derivative of inverse tangent

Use it when you integrate by parts and need the derivative of $\tan^{-1} x$. Used in: Nov 2025 paper 32 Q4, the exact value of the integral of x $\tan^{-1} x$; Jun 2024 paper 31 Q10(b), the integral of $\tan^{-1}(2x)$.

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

GIVEN

Product rule

Label u and v, find du/dx and dv/dx, then substitute. Used in: Nov 2025 paper 31 Q4, a product with exponentials and trigonometric functions; Jun 2025 paper 31 Q11(a), the maximum of a product of trigonometric functions; Jun 2024 paper 31 Q5.

$$y = uv \ \Rightarrow \ \frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

GIVEN

Quotient rule

Label u and v and use the printed quotient rule, or rewrite as a product. Used in: Nov 2025 paper 31 Q7, x as a quotient in t; Jun 2024 paper 31 Q5, y as a quotient with cos x.

$$y = \frac{u}{v} \ \Rightarrow \ \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

GIVEN

Parametric differentiation

Find dy/dt and dx/dt, divide, then substitute the parameter at the point. Used in: Nov 2025 paper 31 Q7, a simplified dy/dx in terms of t; Jun 2025 paper 31 Q4, the tangent at the point (e, 3) of a parametric curve.

$$x = \mathrm{f}(t), \ y = \mathrm{g}(t) \ \Rightarrow \ \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

GIVEN

Chain rule

Differentiate the outer function, then multiply by the derivative of the inside, for e^(sin 2x), cos(2x) and powers of brackets. Used in: Nov 2025 paper 31 Q4, a derivative with e^(sin 2x); Jun 2025 paper 31 Q11(a); Jun 2024 paper 31 Q5.

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

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Implicit differentiation

Differentiate every term with respect to x, put dy/dx on each y term, then collect and solve for dy/dx. Used in: Nov 2025 paper 32 Q7(a), showing an expression for dy/dx; Nov 2025 paper 35 Q4, the gradient at a point.

$$\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}, \qquad \frac{d}{dx}(xy) = y + x\frac{dy}{dx}$$

LEARN IT

Tangent and normal

A normal parallel to the y-axis means a horizontal tangent, so dy/dx = 0. Used in: Jun 2025 paper 31 Q4, the tangent in the form y = mx + c; Nov 2025 paper 32 Q7(b), where the normal is parallel to the y-axis.

$$m_{\text{tangent}} = \frac{dy}{dx}, \qquad m_{\text{normal}} = -\frac{1}{dy/dx}, \qquad y - y_1 = m(x - x_1)$$

LEARN IT

Integration

Integration by parts

Choose u so that du/dx is simpler and dv/dx is easy to integrate; for ln x take u = ln x. Used in: Nov 2025 paper 31 Q1, the exact value of the integral of ln 3x; Jun 2025 paper 31 Q9(a), the integral of x ln x equated to a constant; Nov 2025 paper 32 Q4.

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

GIVEN

One over x squared plus a squared

With a = 1, it is the integral of 1/(1 + x^2) = $\tan^{-1} x$. Used in: Nov 2025 paper 32 Q4, splitting x^2/(1 + x^2) into 1 - 1/(1 + x^2).

$$\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

GIVEN

Integrals of linear functions

Divide by a, the coefficient of x inside, and add + c. MF19 prints only the plain forms. Used in: Nov 2025 paper 32 Q9(b), integrating partial fractions to logarithms; Jun 2025 paper 33 Q3, the exact value of the integral of $\cos^2$ 5x, after a double angle identity.

$$\int \frac{1}{ax+b}dx = \tfrac{1}{a}\ln|ax+b|, \quad \int e^{ax+b}dx = \tfrac{1}{a}e^{ax+b}, \quad \int \cos(ax+b)dx = \tfrac{1}{a}\sin(ax+b)$$

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Volume of revolution

Square y first and keep pi outside until the end; a squared trigonometric function needs a double angle identity before you integrate. Used in: Jun 2025 paper 31 Q11(b), the exact volume when the region is rotated through 2 pi about the x-axis; Nov 2020 paper 32 Q10(c), a volume in terms of pi.

$$V = \pi\int_a^b y^2\,dx \quad \text{(about the } x\text{-axis)}$$

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Numerical solution of equations

Iterative formula

Start from the given value and repeat until the answer is stable to the accuracy asked, giving each iteration to the decimal places stated; keep the calculator in radians. In Nov 2025 paper 31 Q9(bcd) and Nov 2025 paper 32 Q6(c) the formula was printed; in Jun 2025 paper 31 Q9(bc) you had to build it from the equation in part (a). Used in: Nov 2025 paper 32 Q6(c); Nov 2025 paper 31 Q9(bcd); Jun 2025 paper 31 Q9(bc).

$$x_{n+1} = \mathrm{F}(x_n)$$

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Vectors

Vector equation of a line

a is the position vector of a point on the line and d a direction vector. A line through two points A and B uses b - a for d. Used in: Nov 2025 paper 31 Q11(a), the line through A and B; Jun 2025 paper 31 Q8(a), the same for l1.

$$\mathbf{r} = \mathbf{a} + t\mathbf{d}, \qquad \mathbf{d} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$$

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Displacement and midpoint

M is the midpoint of AB. Used in: Nov 2025 paper 32 Q11(a), the vectors MB and MC from a midpoint M; Nov 2024 paper 32 Q9(a), the position vector of D in a trapezium ABCD.

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \qquad \overrightarrow{OM} = \tfrac{1}{2}(\mathbf{a} + \mathbf{b})$$

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Magnitude of a vector

Square each component, add, then take the root. Used in: Nov 2025 paper 32 Q11(b), the moduli in the cosine of an angle; Jun 2025 paper 31 Q8(c), the acute angle between two lines.

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

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Scalar product

For the angle between two lines use their direction vectors; the scalar product is 0 for perpendicular lines. Used in: Nov 2025 paper 32 Q11(b), the cosine of angle CMB; Jun 2025 paper 31 Q8(c), the acute angle between the directions of l1 and l2; Jun 2024 paper 31 Q9(a), l1 perpendicular to l2.

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$$

GIVEN

Area of a triangle from two sides and the angle between

Find sin C from cos C with $\sin^2 C + \cos^2 C = 1$. Used in: Nov 2025 paper 32 Q11(c), the exact area of triangle ABC from angle CMB.

$$\text{area} = \tfrac{1}{2}ab\sin C$$

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Differential equations and modelling

Separable differential equation

Separate the variables, integrate both sides, add one constant c, then use the given values to find c (and k if needed). Used in: Nov 2025 paper 31 Q8, a differential equation with an unknown constant k; Nov 2025 paper 33 Q10(b), a differential equation with cos x; Jun 2025 paper 31 Q10(b), after dividing out a quotient.

$$\frac{dy}{dx} = \mathrm{f}(x)\,\mathrm{g}(y) \;\Rightarrow\; \int \frac{1}{\mathrm{g}(y)}\,dy = \int \mathrm{f}(x)\,dx$$

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Connected rates of change

Write dV/dt as the rate in minus the rate out, find V in terms of h or r, then use the chain rule to get an equation in h or r and t. Used in: Nov 2025 paper 32 Q10(a), the depth of water in a tank; Nov 2024 paper 31 Q10(a), a cylindrical tank; Nov 2024 paper 32 Q10(a), a spherical balloon.

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

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Volume of a sphere

For a balloon in the shape of a sphere, dV/dr = 4 pi r^2. Used in: Nov 2024 paper 32 Q10(a), air pumped in and flowing out of a spherical balloon, giving the equation for r and t.

$$V = \frac{4}{3}\pi r^3$$

GIVEN

Volume of a cylinder

For a tank of radius r, V = pi r^2 h, so dV/dh = pi r^2. Used in: Nov 2024 paper 31 Q10(a), a cylindrical tank of radius 4 metres, V = 16 pi h.

$$V = \pi r^2 h$$

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Complex numbers

Quadratic equation

It works for complex coefficients too: use i^2 = -1, then multiply top and bottom by the conjugate of the denominator to remove i. Used in: Nov 2025 paper 31 Q6, solving a quadratic with complex coefficients.

$$ax^2 + bx + c = 0 \;\Rightarrow\; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

GIVEN

Modulus and conjugate

Write z = x + iy and compare real and imaginary parts to solve. Used in: Jun 2025 paper 31 Q3, a real condition and a given modulus; Nov 2025 paper 35 Q5, an equation containing z and its conjugate; Nov 2025 paper 31 Q6.

$$|z| = \sqrt{x^2 + y^2}, \qquad z^* = x - iy, \qquad zz^* = |z|^2$$

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Polar and exponential form

Give θ in the principal range and in the quadrant that matches the point. Used in: Jun 2024 paper 31 Q4(a), exact values of r and θ; Jun 2025 paper 31 Q6(a), numbers in the form re^(i θ); Nov 2025 paper 31 Q5(b), the least value of arg z in a region.

$$z = r(\cos\theta + i\sin\theta) = re^{i\theta}, \quad r = |z|,\ \theta = \arg z\ (-\pi < \theta \le \pi)$$

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Multiplying and dividing in polar form

Multiplying by a number of modulus k and argument θ enlarges by k and rotates by θ. Used in: Jun 2025 paper 31 Q6(c), the geometric effect of multiplying; Jun 2024 paper 31 Q4(b), uv in the form re^(i θ).

$$|z_1 z_2| = |z_1||z_2|, \quad \arg(z_1 z_2) = \arg z_1 + \arg z_2, \quad \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|},$$
$$\arg\frac{z_1}{z_2} = \arg z_1 - \arg z_2$$

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Loci in an Argand diagram

Inequalities shade the inside or outside; boundaries are included for at least or at most. Used in: Jun 2024 paper 31 Q7(a), a circle and a perpendicular bisector; Nov 2025 paper 31 Q5(a), the inequalities for a shaded region; Nov 2025 paper 32 Q3(a), the point P where a line meets a circle.

$$|z - a| = r\ \text{(circle)}, \quad |z - a| = |z - b|\ \text{(perpendicular bisector)}, \quad \operatorname{Re} z = k\ \text{(vertical line)}$$

LEARN IT

Also learn these

Polynomial division

Divide by a linear factor to leave a quotient and a remainder; the quotient can then be integrated or used in a differential equation. Used in: Jun 2025 paper 31 Q10(a); Nov 2024 paper 32 Q10(bc).

Partial fractions then expand or integrate

Once you have partial fractions, expand each as a binomial series or integrate each to a logarithm. Used in: Nov 2025 paper 31 Q10(b), the expansion up to x^2; Nov 2025 paper 32 Q9(b), the exact value of the integral; Jun 2025 paper 33 Q7(abc).

Linear form with logarithms

Take ln of both sides and use the power law, for example a^y = bx gives y ln a = ln b + ln x, then read the gradient and intercept from the line. Used in: Jun 2024 paper 31 Q3.

Substitution in an integral

Write du in terms of dx, change the limits to the new variable, then integrate. Used in: Jun 2024 paper 31 Q8; Nov 2025 paper 35 Q7(a); Nov 2025 paper 33 Q11(b).

Show a root lies in an interval

Evaluate the rearranged equation at both ends and state that the sign changes. Used in: Nov 2025 paper 32 Q6(b); Jun 2024 paper 31 Q6(b).

Sketching a pair of graphs to show one root

Draw both graphs on one diagram and mark the single intersection to show exactly one root in the interval. Used in: Nov 2025 paper 31 Q9(a); Nov 2025 paper 32 Q6(a); Jun 2024 paper 31 Q6(a).

Stationary points of a function

Set dy/dx = 0 and reduce the equation to one trigonometric function, then solve for x. Used in: Nov 2025 paper 31 Q4; Jun 2025 paper 31 Q11(a); Jun 2024 paper 31 Q5.

Intersection of two lines

Write a general point of each line, equate two pairs of components to find the parameters, then check the third pair: if it fails, the lines do not meet. Used in: Nov 2025 paper 31 Q11(b); Jun 2025 paper 31 Q8(b); Jun 2024 paper 31 Q9(b).

Foot of a perpendicular and reflection

For a general point P on the line, find the vector from the external point to P, set its scalar product with the direction vector to 0, then double the displacement to reflect. Used in: Nov 2025 paper 31 Q11(c); Jun 2025 paper 33 Q9(b); Jun 2024 paper 31 Q9(d).

Forming a differential equation

A rate of change proportional to a quantity gives dx/dt = kx; use the given values to find k, then rearrange to the form stated. Used in: Jun 2024 paper 31 Q11(a); Nov 2025 paper 32 Q10(a); Nov 2024 paper 31 Q10(a).

Separating variables with partial fractions

If one side is a rational function, split it into partial fractions before integrating, then combine the logarithms into one. Used in: Jun 2024 paper 31 Q11(b); Nov 2025 paper 32 Q10(b).

Complex equations

Substitute z = x + iy, equate real and imaginary parts to get two equations, then solve for x and y. Used in: Jun 2025 paper 31 Q3; Nov 2025 paper 35 Q5.

Greatest or least argument and modulus

Find the point of the region at the greatest angle from the real axis using right-angled triangles or a tangent from the origin, and add or subtract the radius to find the least distance between two loci. Used in: Nov 2025 paper 32 Q3(b); Nov 2025 paper 31 Q5(b); Jun 2024 paper 31 Q7(b).

Using the booklet in the exam

- Learn the R-formula, the forms of partial fractions, the factor and remainder theorems, the separable method and the vector line equation. MF19 prints none of them.
- MF19 prints the binomial series for rational n, the standard derivatives, the product, quotient and parametric rules, integration by parts and the scalar product. Use them, but add the chain-rule factor yourself.
- In integration, MF19 gives only plain forms. For e^2x - 5), 1/(2x - 5) or cos 5x, divide by the coefficient of x inside the bracket and write + c.
- For an iterative formula, work in radian mode, give each iteration to the places the question states, and finish by showing a sign change round the answer.
- In a complex number question, write z = x + iy, use i^2 = -1, and compare real and imaginary parts. Give arguments in the range -pi to pi.
- For two lines in three dimensions, equate two pairs of components, solve, and test the third pair before you claim they meet.

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