STUDY FORMULA BOOKLET

Cambridge AS and A Level Maths 9709 and Further Maths 9231 formulae and tables

MF19 gives pure, mechanics and statistics formulae plus statistical tables. Further maths sections are 9231 only; you must still recall the sine rule, cosine rule and mechanics formulae.

IN THE BOOKLET printed in the official booklet LEARN IT not printed: students must recall it

Based on the official **Cambridge International AS & A Level Mathematics (9709) and Further Mathematics (9231): List of formulae and statistical tables, MF19 (CST319), for use from 2020**, with what to learn and exam tips added by CuratedPrep.

Pure mathematics: mensuration

Pure mathematics sections are used by both 9709 and 9231 students.

Volume of a sphere r is the radius.	$V = \frac{4}{3}\pi r^3$	IN THE BOOKLET
Surface area of a sphere	$A = 4\pi r^2$	IN THE BOOKLET
Volume of a cone or pyramid	$V = \frac{1}{3} \times \text{base area} \times \text{height}$	IN THE BOOKLET
Area of the curved surface of a cone r is the radius of the base.	$A = \pi r \times \text{slant height}$	IN THE BOOKLET
Arc length of a circle θ is in radians.	$s = r\theta$	IN THE BOOKLET
Area of a sector of a circle θ is in radians.	$A = \frac{1}{2}r^2\theta$	IN THE BOOKLET

Pure mathematics: algebra

Quadratic equation	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	IN THE BOOKLET
Arithmetic series: nth term a is the first term, d the common difference.	$u_n = a + (n-1)d$	IN THE BOOKLET
Arithmetic series: sum of n terms l is the last term.	$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$	IN THE BOOKLET
Geometric series: nth term r is the common ratio.	$u_n = ar^{n-1}$	IN THE BOOKLET
Geometric series: sum of n terms	$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$	IN THE BOOKLET
Geometric series: sum to infinity Only valid when the series converges.	$S_\infty = \frac{a}{1-r} \quad (\lvert r \rvert < 1)$	IN THE BOOKLET
Binomial series: positive integer n n is a positive integer.	$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \cdots + b^n$	IN THE BOOKLET
Binomial coefficient	$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$	IN THE BOOKLET

Binomial series: rational n n is rational. The expansion is valid only for |x| < 1.	$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \quad (\lvert x\rvert < 1)$	IN THE BOOKLET

Pure mathematics: trigonometry

Tangent	$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$	IN THE BOOKLET
Pythagorean identity	$\cos^2\theta + \sin^2\theta \equiv 1$	IN THE BOOKLET
Tangent and secant identity	$1 + \tan^2\theta \equiv \sec^2\theta$	IN THE BOOKLET
Cotangent and cosecant identity	$\cot^2\theta + 1 \equiv \operatorname{cosec}^2\theta$	IN THE BOOKLET
Sine of a sum or difference	$\sin(A \pm B) \equiv \sin A\cos B \pm \cos A\sin B$	IN THE BOOKLET
Cosine of a sum or difference The signs on the two sides are opposite.	$\cos(A \pm B) \equiv \cos A\cos B \mp \sin A\sin B$	IN THE BOOKLET
Tangent of a sum or difference The signs in the numerator and denominator are opposite.	$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$	IN THE BOOKLET
Double angle: sine	$\sin 2A \equiv 2\sin A\cos A$	IN THE BOOKLET
Double angle: cosine	$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$	IN THE BOOKLET
Double angle: tangent	$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$	IN THE BOOKLET
Principal values of the inverse functions	$-\frac{1}{2}\pi \leqslant \sin^{-1}x \leqslant \frac{1}{2}\pi, \quad 0 \leqslant \cos^{-1}x \leqslant \pi, \quad -\frac{1}{2}\pi < \tan^{-1}x < \frac{1}{2}\pi$	IN THE BOOKLET

Pure mathematics: differentiation

9709 Pure 1 needs x to the power n; Pure 2 adds e to the power x, ln x, sin x, cos x and tan x; the inverse tangent derivative is Pure 3 and Further Pure only. The product, quotient and parametric rules are Pure 2 and 3.

Derivative of x to the power n	$\frac{d}{dx}(x^n) = nx^{n-1}$	IN THE BOOKLET
Derivative of ln x	$\frac{d}{dx}(\ln x) = \frac{1}{x}$	IN THE BOOKLET
Derivative of e to the power x	$\frac{d}{dx}(e^x) = e^x$	IN THE BOOKLET
Derivative of sin x	$\frac{d}{dx}(\sin x) = \cos x$	IN THE BOOKLET
Derivative of cos x	$\frac{d}{dx}(\cos x) = -\sin x$	IN THE BOOKLET
Derivative of tan x	$\frac{d}{dx}(\tan x) = \sec^2 x$	IN THE BOOKLET
Derivative of sec x	$\frac{d}{dx}(\sec x) = \sec x\tan x$	IN THE BOOKLET
Derivative of cosec x	$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x\cot x$	IN THE BOOKLET
Derivative of cot x	$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$	IN THE BOOKLET
Derivative of inverse tangent	$\frac{d}{dx}\left(\tan^{-1}x\right) = \frac{1}{1+x^2}$	IN THE BOOKLET

Product rule	$y = uv \;\Rightarrow\; \dfrac{dy}{dx} = v\dfrac{du}{dx} + u\dfrac{dv}{dx}$	IN THE BOOKLET
Quotient rule	$y = \dfrac{u}{v} \;\Rightarrow\; \dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	IN THE BOOKLET
Parametric differentiation	$x = \mathrm{f}(t),\ y = \mathrm{g}(t) \;\Rightarrow\; \dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	IN THE BOOKLET

Pure mathematics: integration

Arbitrary constants are omitted in the booklet, and a is a positive constant. Always add + c to an indefinite integral.

Power of x	$\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$	IN THE BOOKLET
One over x	$\displaystyle\int \frac{1}{x}\,dx = \ln\lvert x\rvert$	IN THE BOOKLET
e to the power x	$\displaystyle\int e^x\,dx = e^x$	IN THE BOOKLET
sin x	$\displaystyle\int \sin x\,dx = -\cos x$	IN THE BOOKLET
cos x	$\displaystyle\int \cos x\,dx = \sin x$	IN THE BOOKLET
sec squared x	$\displaystyle\int \sec^2 x\,dx = \tan x$	IN THE BOOKLET
One over x squared plus a squared	$\displaystyle\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	IN THE BOOKLET
One over x squared minus a squared	$\displaystyle\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert \quad (x > a)$	IN THE BOOKLET
One over a squared minus x squared	$\displaystyle\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert \quad (\lvert x\rvert < a)$	IN THE BOOKLET
Integration by parts	$\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$	IN THE BOOKLET
Integral of f dash over f	$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,dx = \ln\lvert\mathrm{f}(x)\rvert$	IN THE BOOKLET

Pure mathematics: vectors

Scalar product For a = a1 i + a2 j + a3 k and b = b1 i + b2 j + b3 k. θ is the angle between a and b.	$\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$	IN THE BOOKLET

Further maths: summations

The booklet puts these under Further Pure Mathematics, so they are for 9231 only.

Sum of the first n integers	$\displaystyle\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$	IN THE BOOKLET
Sum of the first n squares	$\displaystyle\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$	IN THE BOOKLET
Sum of the first n cubes	$\displaystyle\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$	IN THE BOOKLET

Further maths: Maclaurin's series

General Maclaurin series	$\mathrm{f}(x) = \mathrm{f}(0) + x\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \cdots + \frac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$	IN THE BOOKLET
Exponential Valid for all x.	$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^r}{r!} + \ldots$	IN THE BOOKLET
Natural logarithm Valid for x greater than -1 and at most 1.	$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{r+1}\frac{x^r}{r} + \ldots$	IN THE BOOKLET
Sine Valid for all x.	$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots$	IN THE BOOKLET
Cosine Valid for all x.	$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots$	IN THE BOOKLET
Inverse tangent Valid for x from -1 to 1 inclusive.	$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots$	IN THE BOOKLET
Hyperbolic sine Valid for all x.	$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2r+1}}{(2r+1)!} + \ldots$	IN THE BOOKLET
Hyperbolic cosine Valid for all x.	$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2r}}{(2r)!} + \ldots$	IN THE BOOKLET
Inverse hyperbolic tangent Valid for x between -1 and 1. Every term is positive.	$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2r+1}}{2r+1} + \ldots$	IN THE BOOKLET

Further maths: trigonometry (t formulae)

Sine in terms of t	$t = \tan\frac{1}{2}x \;\Rightarrow\; \sin x = \frac{2t}{1+t^2}$	IN THE BOOKLET
Cosine in terms of t	$t = \tan\frac{1}{2}x \;\Rightarrow\; \cos x = \frac{1-t^2}{1+t^2}$	IN THE BOOKLET

Further maths: hyperbolic functions

Basic identity	$\cosh^2 x - \sinh^2 x \equiv 1$	IN THE BOOKLET
Double angle: sinh	$\sinh 2x \equiv 2\sinh x \cosh x$	IN THE BOOKLET
Double angle: cosh	$\cosh 2x \equiv \cosh^2 x + \sinh^2 x$	IN THE BOOKLET
Inverse sinh	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$	IN THE BOOKLET
Inverse cosh	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right) \quad (x \geqslant 1)$	IN THE BOOKLET
Inverse tanh	$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad (\lvert x\rvert < 1)$	IN THE BOOKLET

Further maths: differentiation

Derivative of inverse sine	$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$	IN THE BOOKLET
Derivative of inverse cosine	$\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$	IN THE BOOKLET
Derivative of sinh x	$\frac{d}{dx}(\sinh x) = \cosh x$	IN THE BOOKLET

Derivative of cosh x	$\frac{d}{dx}(\cosh x) = \sinh x$	IN THE BOOKLET
Derivative of tanh x	$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$	IN THE BOOKLET
Derivative of inverse sinh	$\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{1+x^2}}$	IN THE BOOKLET
Derivative of inverse cosh	$\frac{d}{dx}\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^2-1}}$	IN THE BOOKLET
Derivative of inverse tanh	$\frac{d}{dx}\left(\tanh^{-1} x\right) = \frac{1}{1-x^2}$	IN THE BOOKLET

Further maths: integration

Arbitrary constants are omitted in the booklet, and a is a positive constant. Always add + c to an indefinite integral.

sec x	$\int \sec x \, dx = \ln\lvert\sec x + \tan x\rvert = \ln\left\lvert\tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right)\right\rvert$ $\left(\lvert x\rvert < \frac{1}{2}\pi\right)$	IN THE BOOKLET
cosec x	$\int \operatorname{cosec} x \, dx = -\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\left\lvert\tan\left(\frac{1}{2}x\right)\right\rvert \quad (0 < x < \pi)$	IN THE BOOKLET
sinh x	$\int \sinh x \, dx = \cosh x$	IN THE BOOKLET
cosh x	$\int \cosh x \, dx = \sinh x$	IN THE BOOKLET
sech squared x	$\int \operatorname{sech}^2 x \, dx = \tanh x$	IN THE BOOKLET
One over root of a squared minus x squared	$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) \quad (\lvert x\rvert < a)$	IN THE BOOKLET
One over root of x squared minus a squared	$\int \frac{1}{\sqrt{x^2-a^2}} \, dx = \cosh^{-1}\left(\frac{x}{a}\right) \quad (x > a)$	IN THE BOOKLET
One over root of a squared plus x squared	$\int \frac{1}{\sqrt{a^2+x^2}} \, dx = \sinh^{-1}\left(\frac{x}{a}\right)$	IN THE BOOKLET

Mechanics: uniformly accelerated motion

The booklet prints only these four. u is initial velocity, v final velocity, a acceleration, t time, s displacement.

First equation	$v = u + at$	IN THE BOOKLET
Second equation	$s = \frac{1}{2}(u+v)t$	IN THE BOOKLET
Third equation	$s = ut + \frac{1}{2}at^2$	IN THE BOOKLET
Fourth equation	$v^2 = u^2 + 2as$	IN THE BOOKLET

Further maths: motion of a projectile

The Further Mechanics sections are for 9231 only.

Equation of the trajectory V is the speed of projection and θ the angle above the horizontal.	$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$	IN THE BOOKLET

Further maths: elastic strings and springs

Tension (Hooke's law) λ is the modulus of elasticity, x the extension, l the natural length.	$T = \dfrac{\lambda x}{l}$	IN THE BOOKLET
Elastic potential energy	$E = \dfrac{\lambda x^2}{2l}$	IN THE BOOKLET

Further maths: motion in a circle

Acceleration in uniform circular motion Directed towards the centre of the circle.	$\omega^2 r$ or $\dfrac{v^2}{r}$	IN THE BOOKLET

Further maths: centres of mass of uniform bodies

Triangular lamina	$\dfrac{2}{3}$ of the way along the median from the vertex	IN THE BOOKLET
Solid hemisphere of radius r	$\dfrac{3}{8}r$ from the centre	IN THE BOOKLET
Hemispherical shell of radius r	$\dfrac{1}{2}r$ from the centre	IN THE BOOKLET
Circular arc of radius r and angle 2 alpha alpha is in radians.	$\dfrac{r\sin\alpha}{\alpha}$ from the centre	IN THE BOOKLET
Circular sector of radius r and angle 2 alpha alpha is in radians.	$\dfrac{2r\sin\alpha}{3\alpha}$ from the centre	IN THE BOOKLET
Solid cone or pyramid of height h	$\dfrac{3}{4}h$ from the vertex	IN THE BOOKLET

Probability and statistics: summary statistics

Probability and statistics sections are used by both 9709 (Papers 5 and 6) and 9231 students.

Ungrouped data: mean	$\bar{x} = \dfrac{\Sigma x}{n}$	IN THE BOOKLET
Ungrouped data: standard deviation	$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$	IN THE BOOKLET
Grouped data: mean	$\bar{x} = \dfrac{\Sigma x f}{\Sigma f}$	IN THE BOOKLET
Grouped data: standard deviation	$\sqrt{\dfrac{\Sigma(x-\bar{x})^2 f}{\Sigma f}} = \sqrt{\dfrac{\Sigma x^2 f}{\Sigma f} - \bar{x}^2}$	IN THE BOOKLET

Probability and statistics: discrete random variables

Expectation	$\mathrm{E}(X) = \Sigma xp$	IN THE BOOKLET
Variance	$\mathrm{Var}(X) = \Sigma x^2 p - \{\mathrm{E}(X)\}^2$	IN THE BOOKLET
Binomial B(n, p): probability	$p_r = \binom{n}{r} p^r (1-p)^{n-r}$	IN THE BOOKLET
Binomial B(n, p): mean and variance	$\mu = np, \quad \sigma^2 = np(1-p)$	IN THE BOOKLET

Geometric Geo(p): probability r = 1, 2, 3, and so on.	$p_r = p(1-p)^{r-1}$	IN THE BOOKLET
Geometric Geo(p): mean The variance is not printed in the booklet.	$\mu = \frac{1}{p}$	IN THE BOOKLET
Poisson Po(λ): probability	$p_r = e^{-\lambda}\frac{\lambda^r}{r!}$	IN THE BOOKLET
Poisson Po(λ): mean and variance	$\mu = \lambda, \quad \sigma^2 = \lambda$	IN THE BOOKLET

Probability and statistics: continuous random variables

Expectation	$\mathrm{E}(X) = \int x\,\mathrm{f}(x)\,dx$	IN THE BOOKLET
Variance	$\mathrm{Var}(X) = \int x^2\,\mathrm{f}(x)\,dx - \{\mathrm{E}(X)\}^2$	IN THE BOOKLET

Probability and statistics: sampling and testing

Unbiased estimate of the mean	$\bar{x} = \frac{\Sigma x}{n}$	IN THE BOOKLET
Unbiased estimate of the variance	$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)$	IN THE BOOKLET
Central Limit Theorem	$\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$	IN THE BOOKLET
Approximate distribution of a sample proportion	$\mathrm{N}\left(p, \frac{p(1-p)}{n}\right)$	IN THE BOOKLET

Further maths: sampling and testing

The Further Probability and Statistics sections are for 9231 only.

Two-sample estimate of a common variance	$s^2 = \frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}$	IN THE BOOKLET

Further maths: probability generating functions

Definition	$\mathrm{G}_X(t) = \mathrm{E}(t^X)$	IN THE BOOKLET
Expectation	$\mathrm{E}(X) = \mathrm{G}'_X(1)$	IN THE BOOKLET
Variance	$\mathrm{Var}(X) = \mathrm{G}''_X(1) + \mathrm{G}'_X(1) - \{\mathrm{G}'_X(1)\}^2$	IN THE BOOKLET

The normal distribution function

If Z has a normal distribution with mean 0 and variance 1, the table gives Phi(z) = P(Z <= z). Read the row for z to one decimal place and the column for the second decimal place. For negative values of z, use Phi(-z) = 1 - Phi(z).

z	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224

z	0	1	2	3	4	5	6	7	8	9
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

The normal distribution function: ADD columns

The booklet prints these nine ADD columns beside the table above. For the third decimal place of z, add the number in the matching column (1 to 9) to the fourth decimal place of Phi(z). Use the same row for z.

z	1	2	3	4	5	6	7	8	9
0.0	4	8	12	16	20	24	28	32	36
0.1	4	8	12	16	20	24	28	32	36
0.2	4	8	12	15	19	23	27	31	35
0.3	4	7	11	15	19	22	26	30	34
0.4	4	7	11	14	18	22	25	29	32
0.5	3	7	10	14	17	20	24	27	31
0.6	3	7	10	13	16	19	23	26	29
0.7	3	6	9	12	15	18	21	24	27
0.8	3	5	8	11	14	16	19	22	25
0.9	3	5	8	10	13	15	18	20	23
1.0	2	5	7	9	12	14	16	19	21
1.1	2	4	6	8	10	12	14	16	18
1.2	2	4	6	7	9	11	13	15	17

z	1	2	3	4	5	6	7	8	9
1.3	2	3	5	6	8	10	11	13	14
1.4	1	3	4	6	7	8	10	11	13
1.5	1	2	4	5	6	7	8	10	11
1.6	1	2	3	4	5	6	7	8	9
1.7	1	2	3	4	4	5	6	7	8
1.8	1	1	2	3	4	4	5	6	6
1.9	1	1	2	2	3	4	4	5	5
2.0	0	1	1	2	2	3	3	4	4
2.1	0	1	1	2	2	2	3	3	4
2.2	0	1	1	1	2	2	2	3	3
2.3	0	1	1	1	1	2	2	2	2
2.4	0	0	1	1	1	1	1	2	2
2.5	0	0	0	1	1	1	1	1	1
2.6	0	0	0	0	1	1	1	1	1
2.7	0	0	0	0	0	1	1	1	1
2.8	0	0	0	0	0	0	0	1	1
2.9	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1, the table gives the value z such that P(Z <= z) = p.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Further maths: critical values for the t-distribution

For 9231 only. If T has a t-distribution with nu degrees of freedom, the table gives the value t such that P(T <= t) = p, for each p (columns) and nu (rows). The last row, infinity, matches the normal critical values.

v	p = 0.75	p = 0.90	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.9975	p = 0.999	p = 0.9995
1	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073

v	p = 0.75	p = 0.90	p = 0.95	p = 0.975	p = 0.99	p = 0.995	p = 0.9975	p = 0.999	p = 0.9995
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Further maths: critical values for the chi-squared distribution

For 9231 only. If X has a chi-squared distribution with nu degrees of freedom, the table gives the value x such that P(X <= x) = p. In the first row the booklet writes $0.0^{3}1571$ for 0.0001571, $0.0^{3}9821$ for 0.0009821 and $0.0^{2}3932$ for 0.003932: the small raised number is the count of zeros after the decimal point.

v	p = 0.01	0.025	0.05	0.9	0.95	0.975	0.99	0.995	0.999
1	$0.0^{3}1571$	$0.0^{3}9821$	$0.0^{2}3932$	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.51
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.647	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79

v	p = 0.01	0.025	0.05	0.9	0.95	0.975	0.99	0.995	0.999
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

Further maths: Wilcoxon signed-rank test

For 9231 only. The sample has size n. P is the sum of the ranks of the positive differences, Q the sum of the ranks of the negative differences, and T is the smaller of P and Q. The table gives the largest value of T that leads to rejection of the null hypothesis at the level shown. 'none' means the booklet leaves the cell empty, so no value of T can be significant at that level for that n. For larger n, each of P and Q can be approximated by a normal distribution with mean n(n + 1)/4 and variance n(n + 1)(2n + 1)/24.

n	One-tailed 0.05 (two-tailed 0.1)	One-tailed 0.025 (two-tailed 0.05)	One-tailed 0.01 (two-tailed 0.02)	One-tailed 0.005 (two-tailed 0.01)
6	2	0	none	none
7	3	2	0	none
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37

Further maths: Wilcoxon rank-sum test, m = 3 to 6

For 9231 only. The two samples have sizes m and n, where m <= n. R_m is the sum of the ranks of the items in the sample of size m, and W is the smaller of R_m and m(n + m + 1) - R_m. The table gives the largest value of W that leads to rejection of the null hypothesis at the level shown. In each heading the first level is for a one-tailed test and the level in brackets is for a two-tailed test.

'none' means the booklet prints a dash, so no value of W can be significant; an empty cell means the booklet leaves it blank (n is smaller than m).

n	m = 3: 0.05 (0.1)	m = 3: 0.025 (0.05)	m = 3: 0.01 (0.02)	m = 4: 0.05 (0.1)	m = 4: 0.025 (0.05)	m = 4: 0.01 (0.02)	m = 5: 0.05 (0.1)	m = 5: 0.025 (0.05)	m = 5: 0.01 (0.02)	m = 6: 0.05 (0.1)	m = 6: 0.025 (0.05)	m = 6: 0.01 (0.02)
3	6	none	none									
4	6	none	none	11	10	none						
5	7	6	none	12	11	10	19	17	16			
6	8	7	none	13	12	11	20	18	17	28	26	24
7	8	7	6	14	13	11	21	20	18	29	27	25
8	9	8	6	15	14	12	23	21	19	31	29	27
9	10	8	7	16	14	13	24	22	20	33	31	28
10	10	9	7	17	15	13	26	23	21	35	32	29

Further maths: Wilcoxon rank-sum test, m = 7 to 10

For larger values of m and n, use a normal distribution with mean m(m + n + 1)/2 and variance mn(m + n + 1)/12 as an approximation to the distribution of R_m.

n	m = 7: 0.05 (0.1)	m = 7: 0.025 (0.05)	m = 7: 0.01 (0.02)	m = 8: 0.05 (0.1)	m = 8: 0.025 (0.05)	m = 8: 0.01 (0.02)	m = 9: 0.05 (0.1)	m = 9: 0.025 (0.05)	m = 9: 0.01 (0.02)	m = 10: 0.05 (0.1)	m = 10: 0.025 (0.05)	m = 10: 0.01 (0.02)
7	39	36	34									
8	41	38	35	51	49	45						
9	43	40	37	54	51	47	66	62	59			
10	45	42	39	56	53	49	69	65	61	82	78	74

Learn these: not in the booklet

9709 P1 equation of a straight line

$y - y_1 = m(x - x_1),\ y = mx + c,\ ax + by + c = 0$

9709 P1 gradient, midpoint and distance

$m = \dfrac{y_2 - y_1}{x_2 - x_1},\ \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right),\ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

9709 P1 parallel and perpendicular lines

parallel: $m_1 = m_2$, perpendicular: $m_1 m_2 = -1$

9709 P1 circle

$(x - a)^2 + (y - b)^2 = r^2$ and $x^2 + y^2 + 2gx + 2fy + c = 0$

9709 P1 discriminant

$b^2 - 4ac > 0$ two real roots, $= 0$ one repeated root, < 0 no real roots

9709 P1 completing the square

$ax^2 + bx + c = a\left(x + \dfrac{b}{2a}\right)^2 + c - \dfrac{b^2}{4a}$

9709 P1 radians

π rad $= 180^\circ$

9709 P1 exact trigonometric values

$\sin 30^\circ = \cos 60^\circ = \frac{1}{2},\ \sin 45^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2},\ \sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2},\ \tan 30^\circ = \frac{1}{\sqrt{3}},\ \tan 45^\circ = 1,\ \tan 60^\circ = \sqrt{3}$

9709 P1 sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

9709 P1 cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

9709 P1 area of a triangle

Area $= \frac{1}{2}ab\sin C$

9709 P1 progression conditions

a, b, c in AP $\iff 2b = a + c$; a, b, c in GP $\iff b^2 = ac$

9709 P1 chain rule

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

9709 P1 stationary points

$\dfrac{dy}{dx} = 0$; $\dfrac{d^2y}{dx^2} > 0$ minimum, $\dfrac{d^2y}{dx^2} < 0$ maximum

9709 P1 tangent and normal gradients

$m_{\text{tangent}} = \dfrac{dy}{dx},\ m_{\text{normal}} = -\dfrac{1}{dy/dx}$

9709 P1 integrating a linear function

$\displaystyle\int (ax + b)^n\,dx = \frac{(ax + b)^{n+1}}{a(n + 1)} \quad (n \neq -1)$

9709 P1 area under a curve

Area $= \displaystyle\int_a^b y\,dx$

9709 P1 volume of revolution

$V = \pi\displaystyle\int_a^b y^2\,dx$ (about the x-axis), $V = \pi\displaystyle\int_c^d x^2\,dy$ (about the y-axis)

9709 P2 factor and remainder theorems

$\mathrm{f}(a) = 0 \iff (x - a)$ is a factor; remainder on dividing by $(ax + b) = \mathrm{f}\left(-\frac{b}{a}\right)$

9709 P2 laws of logarithms

$\log_a x + \log_a y = \log_a(xy),\ \log_a x - \log_a y = \log_a \dfrac{x}{y},\ \log_a x^k = k \log_a x$

9709 P2 logarithms and indices

$a^x = b \iff x = \log_a b;\ \ln e^x = x,\ e^{\ln x} = x$

9709 P2 straight-line form

$y = kx^n \Rightarrow \ln y = \ln k + n \ln x;\ \ y = ka^x \Rightarrow \ln y = \ln k + x \ln a$

9709 P2 reciprocal trigonometric functions

$\sec\theta = \dfrac{1}{\cos\theta},\ \operatorname{cosec}\theta = \dfrac{1}{\sin\theta},\ \cot\theta = \dfrac{1}{\tan\theta}$

9709 P2 modulus

$|x - a| < b \iff a - b < x < a + b;\ \ |a| = |b| \iff a^2 = b^2$

9709 P2 integrals of linear functions

$\displaystyle\int e^{ax+b}dx = \frac{1}{a}e^{ax+b},\ \int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b|,\ \int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b),\ \int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b),\ \int \sec^2(ax+b)dx = \frac{1}{a}\tan(ax+b)$

9709 P2 trapezium rule

$\displaystyle\int_a^b y\,dx \approx \frac{h}{2}\{y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n\},\ h = \frac{b-a}{n}$

9709 P2 iteration

$x_{n+1} = \mathrm{F}(x_n)$ converges to a root of $x = \mathrm{F}(x)$

9709 P3 R-formula

$a\sin\theta + b\cos\theta = R\sin(\theta + \alpha),\ R = \sqrt{a^2 + b^2},\ \tan\alpha = \dfrac{b}{a}$

9709 P3 partial fractions (forms to recall)

$\dfrac{A}{ax+b} + \dfrac{B}{cx+d};\ \dfrac{A}{ax+b} + \dfrac{B}{(cx+d)} + \dfrac{C}{(cx+d)^2};\ \dfrac{A}{ax+b} + \dfrac{Bx+C}{cx^2+d}$

9709 P3 separable differential equation

$\dfrac{dy}{dx} = \mathrm{f}(x)\mathrm{g}(y) \Rightarrow \displaystyle\int \frac{1}{\mathrm{g}(y)}dy = \int \mathrm{f}(x)\,dx$

9709 P3 complex numbers (Cartesian)

$z = x + iy,\ i^2 = -1,\ z^* = x - iy,\ |z| = \sqrt{x^2 + y^2},\ zz^* = |z|^2$

9709 P3 complex numbers (polar)

$z = r(\cos\theta + i\sin\theta) = re^{i\theta},\ r = |z|,\ \theta = \arg z\ (-\pi < \theta \leqslant \pi)$

9709 P3 multiplying and dividing in polar form

$|z_1 z_2| = |z_1||z_2|,\ \arg(z_1 z_2) = \arg z_1 + \arg z_2,\ \left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|},\ \arg\dfrac{z_1}{z_2} = \arg z_1 - \arg z_2$

9709 P3 loci in an Argand diagram

$|z - a| = k$ (circle), $|z - a| = |z - b|$ (perpendicular bisector), $\arg(z - a) = \alpha$ (half-line)

9709 P3 vector line

$\mathbf{r} = \mathbf{a} + t\mathbf{b}$, magnitude $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$, midpoint of $AB = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$

9709 P3 angle between two vectors

$\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

9709 M Newton's second law and weight

$F = ma,\ W = mg$

9709 M friction

$F \leqslant \mu R$, limiting equilibrium: $F = \mu R$

9709 M resolving on a slope

component down the slope $= mg\sin\theta$, component into the slope $= mg\cos\theta$

9709 M calculus in kinematics

$v = \dfrac{ds}{dt},\ a = \dfrac{dv}{dt},\ s = \displaystyle\int v\,dt,\ v = \int a\,dt$

9709 M momentum and collisions

$p = mv,\ m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ (coalescing: $(m_1 + m_2)v$)

9709 M work, energy and power

$W = Fd\cos\theta,\ KE = \frac{1}{2}mv^2,\ GPE = mgh,\ P = Fv$, average power $= \dfrac{\text{work done}}{\text{time taken}}$

9709 M work-energy principle

work done by external forces $= \Delta KE + \Delta PE$

9709 S1 interquartile range

$IQR = Q_3 - Q_1$

9709 S1 coded data

$\bar{x} = a + \dfrac{\Sigma(x-a)}{n}$, standard deviation $= \sqrt{\dfrac{\Sigma(x-a)^2}{n} - \left(\dfrac{\Sigma(x-a)}{n}\right)^2}$

9709 S1 permutations and combinations

${}^nP_r = \dfrac{n!}{(n-r)!},\ {}^nC_r = \dfrac{n!}{r!(n-r)!}$, repeated items: $\dfrac{n!}{p!\,q!}$

9709 S1 conditional probability

$\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

9709 S1 independent and exclusive events

$\text{independent: } \mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B);\ \text{exclusive: } \mathrm{P}(A \cap B) = 0$

9709 S1 addition rule

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$

9709 S1 standardising

$$Z = \frac{X - \mu}{\sigma},\ X \sim \mathrm{N}(\mu, \sigma^2)$$

9709 S1 normal approximation to the binomial

$X \sim \mathrm{B}(n, p) \approx \mathrm{N}(np, npq)$ when $np > 5$ and $nq > 5,$ with a continuity correction of ± 0.5

9709 S2 Poisson as an approximation to the binomial

$\mathrm{B}(n, p) \approx \mathrm{Po}(np)$ when $n > 50$ and $np < 5$

9709 S2 normal approximation to the Poisson

$\mathrm{Po}(\lambda) \approx \mathrm{N}(\lambda, \lambda)$ when $\lambda > 15,$ with a continuity correction

9709 S2 sum of independent Poisson variables

$$X \sim \mathrm{Po}(\lambda_1),\ Y \sim \mathrm{Po}(\lambda_2) \Rightarrow X + Y \sim \mathrm{Po}(\lambda_1 + \lambda_2)$$

9709 S2 linear combinations

$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b,\ \mathrm{Var}(aX + b) = a^2\mathrm{Var}(X),\ \mathrm{E}(aX + bY) = a\mathrm{E}(X) + b\mathrm{E}(Y),\ \mathrm{Var}(aX + bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$ (independent)

9709 S2 probability density function

$\mathrm{f}(x) \geqslant 0,\ \displaystyle\int_{-\infty}^{\infty} \mathrm{f}(x)\,dx = 1,\ \mathrm{P}(a < X < b) = \int_a^b \mathrm{f}(x)\,dx,\ \text{median } m: \int_{-\infty}^{m} \mathrm{f}(x)\,dx = \tfrac{1}{2}$

9709 S2 mean and variance of a sample mean

$$\mathrm{E}(\bar{X}) = \mu,\ \mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$$

9709 S2 confidence interval for a mean

$$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}\ \ (z = 1.645,\ 1.960,\ 2.576 \text{ for } 90\%,\ 95\%,\ 99\%)$$

9709 S2 confidence interval for a proportion

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

9709 S2 test statistic for a mean

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

9709 S2 errors in hypothesis tests

Type I: reject H_0 when it is true; $\mathrm{P}(\text{Type I}) =$ significance level; Type II: accept H_0 when it is false

9231 FP1 roots of a quadratic

$$\alpha + \beta = -\frac{b}{a},\ \alpha\beta = \frac{c}{a}$$

9231 FP1 roots of a cubic $ax^3+bx^2+cx+d=0$

$$\sum \alpha = -\frac{b}{a},\ \sum \alpha\beta = \frac{c}{a},\ \alpha\beta\gamma = -\frac{d}{a}$$

9231 FP1 roots of a quartic $ax^4+bx^3+cx^2+dx+e=0$

$$\sum \alpha = -\frac{b}{a},\ \sum \alpha\beta = \frac{c}{a},\ \sum \alpha\beta\gamma = -\frac{d}{a},\ \alpha\beta\gamma\delta = \frac{e}{a}$$

9231 FP1 symmetric functions

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

9231 FP1 determinant and inverse of a 2 by 2 matrix

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc,\ \ \mathbf{M}^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

9231 FP1 inverse of a product

$$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

9231 FP1 area scale factor

$$\text{area scale factor} = |\det \mathbf{M}|$$

9231 FP1 rotation matrix, anticlockwise about the origin through angle θ

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$9231FP1reflectionintheliney = x\tan\theta:$
$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

9231 FP1 enlargement, stretch and shear

enlargement, scale factor k: $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$; stretch parallel to the x-axis: $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$; shear, x-axis invariant: $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$

9231 FP1 polar coordinates

$$x = r\cos\theta,\ y = r\sin\theta,\ r^2 = x^2 + y^2,\ \tan\theta = \frac{y}{x}$$

9231 FP1 area of a sector in polar form

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2\,d\theta$$

9231 FP1 equations of a plane

$$ax + by + cz = d,\ \ \mathbf{r}\cdot\mathbf{n} = p,\ \ \mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

9231 FP1 vector product

$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$

9231 FP1 angle between a line and a plane

$$\sin\theta = \frac{|\mathbf{b}\cdot\mathbf{n}|}{|\mathbf{b}||\mathbf{n}|};\ \ \text{between two planes: } \cos\theta = \frac{|\mathbf{n}_1\cdot\mathbf{n}_2|}{|\mathbf{n}_1||\mathbf{n}_2|}$$

9231 FP1 shortest distance between skew lines

$$d = \frac{|(\mathbf{a}_2 - \mathbf{a}_1)\cdot(\mathbf{b}_1\times\mathbf{b}_2)|}{|\mathbf{b}_1\times\mathbf{b}_2|}$$

9231 FP2 definitions of the hyperbolic functions

$$\sinh x = \frac{e^x - e^{-x}}{2},\ \cosh x = \frac{e^x + e^{-x}}{2},\ \tanh x = \frac{\sinh x}{\cosh x}$$

9231 FP2 reciprocal hyperbolic functions

$$\operatorname{sech} x = \frac{1}{\cosh x},\ \operatorname{cosech} x = \frac{1}{\sinh x},\ \coth x = \frac{1}{\tanh x}$$

9231 FP2 hyperbolic identities

$$1 - \tanh^2 x \equiv \operatorname{sech}^2 x,\ \ \coth^2 x - 1 \equiv \operatorname{cosech}^2 x$$

9231 FP2 hyperbolic addition formulae

$\sinh(A \pm B) \equiv \sinh A \cosh B \pm \cosh A \sinh B$, $\cosh(A \pm B) \equiv \cosh A \cosh B \pm \sinh A \sinh B$

9231 FP2 derivatives of the other hyperbolic functions

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$, $\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$, $\frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$

9231 FP2 eigenvalues and eigenvectors

$\mathbf{Ae} = \lambda\mathbf{e}$, $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$ (characteristic equation)

9231 FP2 diagonalisation

$\mathbf{A} = \mathbf{QDQ}^{-1}$, $\mathbf{A}^n = \mathbf{QD}^n\mathbf{Q}^{-1}$

9231 FP2 Cayley-Hamilton theorem

a square matrix satisfies its own characteristic equation

9231 FP2 second derivative of a parametric curve

$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$

9231 FP2 arc length (Cartesian, parametric, polar)

$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$, $s = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$, $s = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$

9231 FP2 surface area of revolution about the x-axis

$S = 2\pi \int y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$, $S = 2\pi \int y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

9231 FP2 surface area of revolution about the y-axis

$S = 2\pi \int x\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$, $S = 2\pi \int x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

9231 FP2 de Moivre's theorem

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, $e^{i\theta} = \cos\theta + i\sin\theta$

9231 FP2 nth roots of unity

$z^n = 1 \Rightarrow z = e^{2\pi ik/n}$, $k = 0, 1, \ldots, n-1$

9231 FP2 trigonometric results from z

$z = \cos\theta + i\sin\theta \Rightarrow z^n + \frac{1}{z^n} = 2\cos n\theta$, $z^n - \frac{1}{z^n} = 2i\sin n\theta$

9231 FP2 integrating factor

$\frac{dy}{dx} + \mathrm{P}(x)y = \mathrm{Q}(x)$, $\mathrm{IF} = e^{\int \mathrm{P}\,dx}$, $y \times \mathrm{IF} = \int \mathrm{Q} \times \mathrm{IF}\,dx$

9231 FP2 second order linear equations, solutions from the auxiliary equation

for $ay'' + by' + cy = 0$, $am^2 + bm + c = 0$; distinct real roots: $y = Ae^{m_1x} + Be^{m_2x}$; repeated root: $y = (A + Bx)e^{mx}$; complex roots $p \pm iq$: $y = e^{px}(A\cos qx + B\sin qx)$

9231 FP2 general solution

$y =$ complementary function $+$ particular integral

9231 FP2 particular integrals to try

polynomial: polynomial of the same degree; ae^{bx} : ke^{bx}; $a\cos px + b\sin px$: $\lambda\cos px + \mu\sin px$; multiply by x if it is in the complementary function

9231 FM projectile motion

$x = Vt\cos\theta$, $y = Vt\sin\theta - \frac{1}{2}gt^2$, $v_y = V\sin\theta - gt$

9231 FM projectile time of flight, greatest height, range

$T = \frac{2V\sin\theta}{g}$, $H = \frac{V^2\sin^2\theta}{2g}$, $R = \frac{V^2\sin 2\theta}{g}$

9231 FM moment of a force

moment $=$ force $\times$ perpendicular distance from the pivot

9231 FM equilibrium of a rigid body

sum of forces $= 0$ and sum of moments about any point $= 0$

9231 FM centre of mass of a system of particles

$\bar{x} = \frac{\sum m_i x_i}{\sum m_i}$

9231 FM angular speed

$v = r\omega$, $T = \frac{2\pi}{\omega}$, force towards the centre $= mr\omega^2 = \frac{mv^2}{r}$

9231 FM vertical circle

energy: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mgh$; string slack when $T = 0$; complete circle needs $T \geqslant 0$ at the top

9231 FM variable force

$F = ma = m\frac{dv}{dt} = mv\frac{dv}{dx}$

9231 FM Newton's experimental law

$e = \frac{\text{speed of separation}}{\text{speed of approach}}$, $0 \leqslant e \leqslant 1$ ($e = 1$ perfectly elastic, $e = 0$ inelastic)

9231 FM direct and oblique impact

conserve momentum along the line of centres; apply e along the line of centres; the component perpendicular to it is unchanged

9231 FM impact with a fixed surface

speed after $= e \times$ speed before (normal component)

9231 FS expectation of a function

$\mathrm{E}(\mathrm{g}(X)) = \int \mathrm{g}(x)\mathrm{f}(x)\,dx$

9231 FS cumulative distribution function

$$\mathrm{F}(x) = \int_{-\infty}^{x} \mathrm{f}(t)\,dt, \quad \mathrm{f}(x) = \frac{d\mathrm{F}}{dx}, \quad \mathrm{P}(a < X < b) = \mathrm{F}(b) - \mathrm{F}(a), \quad \text{median } m : \mathrm{F}(m) = \tfrac{1}{2}$$

9231 FS t-test for one mean

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}, \quad \nu = n - 1$$

9231 FS confidence interval for a mean (small sample)

$$\bar{x} \pm t_{\nu} \frac{s}{\sqrt{n}}$$

9231 FS two-sample t-test

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \quad \nu = n_1 + n_2 - 2, \quad s^2 \text{ is the pooled estimate}$$

9231 FS paired t-test

$$t = \frac{\bar{d}}{s_d/\sqrt{n}}, \quad \nu = n - 1$$

9231 FS difference of means (known variances)

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

9231 FS chi-squared statistic

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

9231 FS degrees of freedom for chi-squared tests

$$\text{goodness of fit: } \nu = \text{cells} - 1 - \text{parameters estimated; contingency table: } \nu = (r - 1)(c - 1)$$

9231 FS expected frequency in a contingency table

$$E = \frac{\text{row total} \times \text{column total}}{\text{grand total}} \quad (\text{combine classes so } E \geqslant 5)$$

9231 FS sign test

$$X \sim \mathrm{B}(n, \tfrac{1}{2}) \text{ under } H_0$$

9231 FS Wilcoxon tests

$$\text{signed-rank: } T = \min(P, Q); \text{ rank-sum: } W = \min(R_m,\ m(n + m + 1) - R_m)$$

9231 FS probability generating function of a distribution

$$\mathrm{G}_X(t) = \sum p_r t^r, \quad \mathrm{G}_X(1) = 1$$

9231 FS PGFs of standard distributions

$$\mathrm{B}(n, p) : (1 - p + pt)^n; \quad \mathrm{Geo}(p) : \frac{pt}{1 - (1 - p)t}; \quad \mathrm{Po}(\lambda) : e^{\lambda(t-1)}$$

9231 FS PGF of a sum of independent variables

$$\mathrm{G}_{X+Y}(t) = \mathrm{G}_X(t)\,\mathrm{G}_Y(t)$$

Using the booklet in the exam

- If you take 9709 only, ignore every section and table labelled Further maths; the t, chi-squared and Wilcoxon tables are 9231 only.
- The booklet gives sin(A plus or minus B), cos(A plus or minus B) and the double angle identities, but not the sine rule, cosine rule or the area of a triangle formula.
- Mechanics prints only the four constant acceleration equations. Forces, friction, momentum, work, energy and power must come from memory.
- In the normal table, read z to two decimal places using the ADD columns for the third decimal place, and use the symmetry rule for negative z.
- The t, chi-squared and normal critical value tables give P(T <= t) = p (lower tail), so for a one-tailed upper test use p = 1 minus the significance level.

Based on: Cambridge International AS & A Level Mathematics 9709, List of formulae and statistical tables (MF19), printed in the 9709 syllabus for 2026 and 2027; Cambridge International AS & A Level Further Mathematics 9231, List of formulae and statistical tables (MF19), printed in the 9231 syllabus for 2026 and 2027. Statistical tables are calculated by CuratedPrep to the same layout. Updated 31 October 2026. Always practise with the version of the official booklet your exam uses. More free sheets and topic worksheets at curatedprep.com.