STUDY FORMULA BOOKLET

Edexcel IAL Maths Unit FP3 (WFM03): further pure mathematics 3

The booklet prints the hyperbolic identities, derivatives and integrals, arc length and surface area, a conics table and the vector formulae. Learn first the exponential definitions, the 3 by 3 determinant and inverse, eigenvalue and diagonalisation rules, angle formulae and the skew lines distance.

GIVEN printed in the formula booklet LEARN IT not in the booklet: learn it

Built by CuratedPrep from 12 past question papers for Unit FP3 in our question bank, January 2021 to January 2026. Given means it is in the Mathematical Formulae and Statistical Tables booklet (Issue 2, 2021) handed out in every unit exam. Learn it means a past question needed it and the exam did not print it.

Hyperbolic functions

Definitions in terms of exponentials

The booklet prints identities but not these definitions, and questions say use the definitions without printing them. Used in: Jun 2025 Q1(a); Jan 2026 Q3(a); Jan 2024 Q5(a).

$$\cosh x = \tfrac{1}{2}(e^x + e^{-x}), \quad \sinh x = \tfrac{1}{2}(e^x - e^{-x}), \quad \tanh x = \frac{\sinh x}{\cosh x}, \quad \operatorname{sech} x = \frac{1}{\cosh x}$$

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Basic identity

Rearranged as $\sinh^2 x = \cosh^2 x - 1$ to turn an equation into a quadratic in cosh x. Used in: Jan 2026 (A) Q1; Jan 2025 Q1; Jan 2022 Q1(a).

$$\cosh^2 x - \sinh^2 x \equiv 1$$

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sinh of a double angle

Also used with 2x or 4x in place of x. Used in: Jan 2026 Q5(a); Jan 2021 Q2(a).

$$\sinh 2x \equiv 2 \sinh x \cosh x$$

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cosh of a double angle

With the basic identity it gives $\cosh 2x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$. Used in: Jan 2026 (A) Q1; Jan 2022 Q1(a).

$$\cosh 2x \equiv \cosh^2 x + \sinh^2 x$$

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Addition formulae

Not on the sheet: the question printed it. Both questions that needed these printed them as a result to prove from the exponential definitions in an earlier part. Used in: Jun 2024 Q4(b), writing 10 sinh x + 8 cosh x as R sinh(x + alpha); Jun 2022 Q1(b).

$$\sinh(A + B) \equiv \sinh A \cosh B + \cosh A \sinh B, \quad \cosh(A + B) \equiv \cosh A \cosh B + \sinh A \sinh B$$

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Tanh and sech identity

One question asked you to prove it from the definitions; the others needed it without printing it. Used in: Jan 2024 Q5(a), proving it; Jan 2021 Q2(a), differentiating ln(tanh 2x).

$$1 - \tanh^2 x \equiv \operatorname{sech}^2 x$$

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Inverse hyperbolic cosine

Use it to give answers as natural logarithms: cosh x = k gives x = plus or minus $\ln(k + \sqrt{k^2 - 1})$. Used in: Jan 2025 Q1; Jan 2022 Q1(b); Jun 2025 Q1(b).

$$\operatorname{arcosh} x \equiv \ln\{x + \sqrt{x^2 - 1}\} \quad (x \geqslant 1)$$

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Inverse hyperbolic sine

$\sinh x = k$ gives $x = \ln(k + \sqrt{k^2 + 1})$. Used in: Jan 2026 (A) Q1.

$$\operatorname{arsinh} x \equiv \ln\{x + \sqrt{x^2 + 1}\}$$

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Ellipse and hyperbola

The conics table below gives the standard and parametric forms, eccentricity, foci, directrices and asymptotes.

Tangent and normal at a point

For a point (a cos θ, b sin θ) or (a cosh t, b sinh t), find m by implicit or parametric differentiation. Used in: Jan 2026 Q8(a); Jun 2024 Q6(a); Jun 2025 Q9(b).

$$y - y_1 = m(x - x_1), \qquad m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}}$$

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Parametric differentiation

Used when the curve is given as x = a cos θ, y = b sin θ or with hyperbolic parameters. Used in: Jan 2026 (A) Q3(a); Jan 2025 Q7(a); Jun 2023 Q6(a).

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

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Midpoint of a line segment

For the midpoint of a chord, or the centre of a circle on a diameter. Used in: Jun 2025 Q9(e), the circle with QR as diameter; Jun 2022 Q9(b).

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

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FP3 Conics

Ellipse, parabola, hyperbola and rectangular hyperbola.

	Ellipse	Parabola	Hyperbola	Rectangular hyperbola
Standard form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric form	$(a\cos\theta, b\sin\theta)$	$(at^2, 2at)$	$(a\sec\theta, b\tan\theta)$; $(\pm a\cosh\theta, b\sinh\theta)$	$\left(ct, \frac{c}{t}\right)$
Eccentricity	$e < 1$; $b^2 = a^2(1 - e^2)$	$e = 1$	$e > 1$; $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(a, 0)$	$(\pm ae, 0)$	$(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices	$x = \pm\frac{a}{e}$	$x = -a$	$x = \pm\frac{a}{e}$	$x + y = \pm\sqrt{2}c$
Asymptotes	none	none	$\frac{x}{a} = \pm\frac{y}{b}$	$x = 0, y = 0$

Differentiation

Derivative of sinh x

Used with the chain rule for arc length and tangent questions. Used in: Jan 2026 Q5(a); Jan 2026 (A) Q3(a).

$$\frac{d}{dx}(\sinh x) = \cosh x$$

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Derivative of cosh x

Derivative of cosh x. Used in: Jan 2026 (A) Q6(a).

$$\frac{d}{dx}(\cosh x) = \sinh x$$

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Derivative of tanh x

For ln(tanh 2x) use the chain rule. Used in: Jan 2021 Q2(a).

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

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Derivative of sech x

Not in the booklet; derive it from sech x = 1/cosh x. Used in: Jan 2024 Q7(a); Jan 2022 Q3(a).

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

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Derivative of arccos x

Derivative of arccos x. Used in: Oct 2021 Q8(a); Jan 2024 Q7(a).

$$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1 - x^2}}$$

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Derivative of arctan x

Derivative of arctan x, with the chain rule for arctan(e^{2x}). Used in: Jun 2021 Q4; Jun 2024 Q3(b).

$$\frac{d}{dx}(\arctan x) = \frac{1}{1 + x^2}$$

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Derivative of arsinh x Derivative of arsinh x. Used in: Jun 2025 Q4(a); Jun 2024 Q3(b).	$\frac{d}{dx}(\operatorname{arsinh} x) = \frac{1}{\sqrt{1+x^2}}$	GIVEN
Derivative of arcosh x Derivative of arcosh x, with the chain rule for arcosh 5x. Used in: Jun 2023 Q8(a); Oct 2021 Q1.	$\frac{d}{dx}(\operatorname{arcosh} x) = \frac{1}{\sqrt{x^2-1}}$	GIVEN
Derivative of artanh x Derivative of artanh x. Used in: Jan 2026 Q9(a).	$\frac{d}{dx}(\operatorname{artanh} x) = \frac{1}{1-x^2}$	GIVEN

Integration, reduction formulae, arc length and surface area

Integral of cosh x For cosh kx divide by k. Used in: Jan 2026 Q5(b).	$\int \cosh x \, dx = \sinh x$	GIVEN
Integral of 1 over (a squared plus x squared) Complete the square on the denominator first. Used in: Jan 2026 (A) Q2(a).	$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$	GIVEN
Integral of 1 over root (a squared plus x squared) Integral of 1/sqrt(a^2 + x^2. Used in: Jan 2026 Q9(b).	$\int \frac{1}{\sqrt{a^2+x^2}} dx = \operatorname{arsinh}\left(\frac{x}{a}\right),\ \ln\{x+\sqrt{x^2+a^2}\}$	GIVEN
Integration by parts The usual first step in a reduction formula: take the power as u. Used in: Jan 2026 (A) Q4(a); Jan 2026 Q7(a); Jun 2023 Q7(a).	$\int u\frac{dv}{dx} dx = uv - \int v\frac{du}{dx} dx$	GIVEN
Tan and sec identity Replace tan^2 x by sec^2 x - 1 to get back to the same integral with a smaller power. Used in: Jan 2026 (A) Q4(a); Jan 2026 Q7(a); Jun 2021 Q5(a).	$1+\tan^2 x \equiv \sec^2 x$	LEARN IT
Arc length (cartesian coordinates) Simplify 1 + (dy/dx)^2 to a perfect square before integrating. Used in: Jun 2024 Q7(a); Jan 2024 Q8(a), the perimeter of a region; Jun 2023 Q3(a).	$s = \int \sqrt{1+\left(\frac{dy}{dx}\right)^2} dx$	GIVEN
Arc length (parametric form) Use it when x and y both depend on t. Used in: Jan 2026 Q5(b); Jan 2025 Q5(b).	$s = \int \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} dt$	GIVEN
Surface area about the x-axis (cartesian) The surface formed by rotating about the x-axis. Used in: Jun 2025 Q7(a); Jun 2021 Q2(b).	$S_x = 2\pi\int y\,ds = 2\pi\int y\sqrt{1+\left(\frac{dy}{dx}\right)^2} dx$	GIVEN
Surface area about the x-axis (parametric form) Use 1 + sinh^2 t = cosh^2 t to make the square root disappear. Used in: Jan 2026 Q5(c); Jan 2025 Q5(c); Jan 2022 Q2.	$S_x = 2\pi\int y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} dt$	GIVEN

Vectors

Vector product The result is perpendicular to both vectors. Used in: Jan 2026 Q4(a); Jun 2025 Q8(a); Oct 2021 Q5(a), a vector perpendicular to two lines.	$\mathbf{a}\times\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta\,\hat{\mathbf{n}}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\a_1&a_2&a_3\\b_1&b_2&b_3\end{vmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}$	GIVEN
Scalar triple product The size of the result is the volume of the parallelepiped. Used in: Jan 2026 Q4(b).	$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=\begin{vmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{vmatrix}=\mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})=\mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})$	GIVEN
Area of a triangle from vectors Use two sides from the same vertex. Used in: Jan 2021 Q1(a), the exact area of a triangle.	$\text{area of triangle } ABC=\frac{1}{2}\left\lvert\overrightarrow{AB}\times\overrightarrow{AC}\right\rvert$	LEARN IT
Volume of a tetrahedron a, b and c are three edges from one vertex, so subtract coordinates first unless a vertex is the origin. Used in: Jan 2026 (A) Q7(a); Jan 2026 Q4(c); Jan 2024 Q6(c).	$V=\frac{1}{6}\lvert\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\rvert$	LEARN IT
Cartesian equation of a line Read off a point on the line and the direction vector from the denominators. Used in: Jan 2022 Q7; Jan 2021 Q7.	$\frac{x-a_1}{b_1}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}\,(=\lambda)$	GIVEN
Cartesian equation of a plane Take n from a vector product and d from a point on the plane. Used in: Jan 2026 (A) Q7(b); Jan 2024 Q6(a); Jun 2021 Q6(a).	$n_1x+n_2y+n_3z+d=0,\quad d=-\mathbf{a}\cdot\mathbf{n}$	GIVEN
Plane through a, parallel to b and c A plane given this way has normal b x c. Used in: Jun 2025 Q8(b); Jun 2024 Q9(a); Oct 2021 Q5(b)(i).	$\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$	GIVEN
Perpendicular distance from (alpha, beta, gamma) to a plane The shortest distance from a point to a plane. Used in: Jan 2026 Q4(d); Jan 2025 Q6(c); Jan 2021 Q7(b).	$\frac{\lvert n_1\alpha+n_2\beta+n_3\gamma+d\rvert}{\sqrt{n_1^2+n_2^2+n_3^2}}$	GIVEN
Angle between two lines or two planes Use the direction vectors for lines and the normals for planes; the modulus gives the acute angle. Used in: Jan 2022 Q7(c), two lines; Jun 2021 Q6(d), two planes.	$\cos\theta=\frac{\lvert\mathbf{a}\cdot\mathbf{b}\rvert}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$	LEARN IT
Angle between a line and a plane b is the direction of the line and n the normal of the plane. Used in: Jan 2025 Q6(b); Jan 2022 Q7(b).	$\sin\theta=\frac{\lvert\mathbf{b}\cdot\mathbf{n}\rvert}{\lvert\mathbf{b}\rvert\lvert\mathbf{n}\rvert}$	LEARN IT
Shortest distance between skew lines a1 and a2 are points on the lines and b1 and b2 their directions. Used in: Jun 2025 Q8(d); Oct 2021 Q5(c).	$d=\frac{\lvert(\mathbf{a}_2-\mathbf{a}_1)\cdot(\mathbf{b}_1\times\mathbf{b}_2)\rvert}{\lvert\mathbf{b}_1\times\mathbf{b}_2\rvert}$	LEARN IT

Further matrix algebra

Determinant of a 3 by 3 matrix

Expand along a row or column, watching the alternating signs. The matrix is singular when it is 0. Used in: Jan 2026 (A) Q5(a); Jun 2025 Q6(a); Oct 2021 Q4(a).

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

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Inverse of a 3 by 3 matrix

Find the matrix of minors, apply the checkerboard signs, transpose, then divide by the determinant. Used in: Jan 2026 (A) Q5(a); Jun 2025 Q6(b); Jan 2026 Q2(b).

$$\mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \operatorname{adj} \mathbf{A}, \quad \operatorname{adj} \mathbf{A} = (\text{matrix of cofactors})^T$$

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Volume scale factor

Used when a matrix transforms a solid. Used in: Oct 2021 Q4(c).

$$\text{volume of image} = |\det \mathbf{M}| \times \text{volume of object}$$

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Eigenvalue equation

Multiply M by a given eigenvector and compare with the vector to read off the eigenvalue. Used in: Jan 2026 Q6(a); Jun 2025 Q3(a); Jan 2025 Q4(a).

$$\mathbf{M}\mathbf{x} = \lambda \mathbf{x}$$

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Characteristic equation

Solve the cubic for the other eigenvalues once one is known, or set it to 0 with a given eigenvalue to find a constant. Used in: Jan 2026 Q6(b); Jun 2023 Q5; Jan 2022 Q4(a).

$$\det(\mathbf{M} - \lambda \mathbf{I}) = 0$$

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Normalised eigenvector

Divide the eigenvector by its length. Used in: Jan 2025 Q4(d); Jun 2024 Q2(a); Jan 2022 Q4(c).

$$\hat{\mathbf{x}} = \frac{\mathbf{x}}{|\mathbf{x}|}$$

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Orthogonal diagonalisation

The columns of P are normalised eigenvectors, in the same order as the eigenvalues on the diagonal of D. Used in: Jan 2025 Q4(e); Jan 2024 Q4(c); Jun 2022 Q3(c).

$$\mathbf{P}^T\mathbf{M}\mathbf{P} = \mathbf{D}, \quad \mathbf{P}^T\mathbf{P} = \mathbf{I}$$

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Image under the inverse

To find the image of a plane, write x, y and z in terms of u, v and w and substitute into the plane's equation. Used in: Jun 2023 Q2(b).

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

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Also learn these

Solving hyperbolic equations

Either replace each term by exponentials and solve a quadratic in e^x, or use identities to get a quadratic in sinh x or cosh x; cosh x is never below 1. Used in: Jan 2026 (A) Q1; Jan 2025 Q1; Jun 2023 Q1.

Proving a hyperbolic identity

Write each side in terms of e^x and e^{-x} and expand; start from one side only. Used in: Jan 2026 Q3(a); Jun 2025 Q1(a); Jun 2022 Q1(a).

Line meeting an ellipse

Substitute y = kx + c into the ellipse to get a quadratic in x; the midpoint of the chord uses the sum of the roots. Used in: Jun 2022 Q9(a); Jun 2022 Q9(b).

Reduction formulae

Integrate by parts, use an identity to write the new integral in terms of I_n and I_(n-2), rearrange, then work down to I_0 or I_1 and substitute the limits. Used in: Jun 2025 Q5; Jan 2026 Q7(a); Jun 2021 Q5(b).

Hyperbolic and trigonometric substitutions

The substitution is usually given; replace dx and the limits, simplify with an identity, then integrate. Used in: Jun 2025 Q7(b); Oct 2021 Q8(c).

Line of intersection of two planes

Find a point on both planes by choosing a value for one coordinate, and take the direction as the cross product of the two normals. Used in: Jun 2025 Q8(c); Jun 2024 Q9(b); Jun 2023 Q4(a).

Foot of a perpendicular and reflections

Write the line through the point in the direction of the normal, substitute into the plane to find the foot, and double the step to reflect. Used in: Jan 2026 (A) Q7(c); Jun 2022 Q8(a); Jan 2024 Q6(b).

Image of a plane or line under a matrix

Apply M to a general point, or use M inverse to write x, y and z in terms of the image coordinates, then substitute. Used in: Jan 2026 (A) Q5(b); Jun 2023 Q2(c); Jun 2024 Q2(b).

Using the booklet in the exam

- Write the exponential definitions of sinh x and cosh x on your paper first: the booklet prints identities and inverse logarithms but not the definitions.
- For a reduction formula, integrate by parts and use an identity to get I_n and I_(n-2), then check the printed answer before you find the base case.
- In arc length and surface area questions, simplify $1 + (dy/dx)^2$ to a perfect square with a hyperbolic or trigonometric identity before you integrate.
- For vectors, get the normal from a cross product, then use the plane formula or the distance formula. Subtract coordinates for edges when the origin is not a vertex.
- For eigenvectors, normalise last, put the columns of P in the same order as the eigenvalues in D, and check that P^T P = I.

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