STUDY FORMULA BOOKLET

IB Maths analysis and approaches (SL and HL) formula booklet

The AA booklet prints formulae for all five topics, with HL only parts, but not exact trig values, tangent equations or binomial probability. Learn those.

IN THE BOOKLET printed in the official booklet LEARN IT not printed: students must recall it

Based on the official **Mathematics: analysis and approaches formula booklet, IB Diploma Programme, version 1.0, first examinations 2021**, with what to learn and exam tips added by CuratedPrep.

Prior learning: SL and HL

Printed in the booklet as prior learning for both levels.

Area of a parallelogram b is the base, h the perpendicular height.	$A = bh$	IN THE BOOKLET
Area of a triangle b is the base, h the perpendicular height.	$A = \frac{1}{2}bh$	IN THE BOOKLET
Area of a trapezoid a and b are the parallel sides, h is the height.	$A = \frac{1}{2}(a+b)h$	IN THE BOOKLET
Area of a circle r is the radius.	$A = \pi r^2$	IN THE BOOKLET
Circumference of a circle r is the radius.	$C = 2\pi r$	IN THE BOOKLET
Volume of a cuboid l is the length, w the width, h the height.	$V = lwh$	IN THE BOOKLET
Volume of a cylinder r is the radius, h the height.	$V = \pi r^2 h$	IN THE BOOKLET
Volume of a prism A is the area of cross-section, h the height.	$V = Ah$	IN THE BOOKLET
Curved surface area of a cylinder r is the radius, h the height.	$A = 2\pi rh$	IN THE BOOKLET
Distance between two points Points (x1, y1) and (x2, y2) in two dimensions.	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$	IN THE BOOKLET
Midpoint of a line segment Endpoints (x1, y1) and (x2, y2) in two dimensions.	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	IN THE BOOKLET

Topic 1: Number and algebra, SL and HL

nth term of an arithmetic sequence u1 is the first term, d the common difference.	$u_n = u_1 + (n-1)d$	IN THE BOOKLET

Sum of n terms of an arithmetic sequence Use the second form when you know the last term.	$S_n = \frac{n}{2}\big(2u_1 + (n-1)d\big); \quad S_n = \frac{n}{2}(u_1 + u_n)$	IN THE BOOKLET
nth term of a geometric sequence r is the common ratio.	$u_n = u_1 r^{\,n-1}$	IN THE BOOKLET
Sum of n terms of a finite geometric sequence Choose the form with a positive denominator.	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$	IN THE BOOKLET
Compound interest FV future value, PV present value, n years, k compounding periods per year, r% nominal annual rate.	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$	IN THE BOOKLET
Exponents and logarithms	$a^x = b \iff x = \log_a b, \quad a > 0,\ b > 0,\ a \neq 1$	IN THE BOOKLET
Laws of logarithms Needs a, x, y > 0.	$\log_a xy = \log_a x + \log_a y; \quad \log_a \frac{x}{y} = \log_a x - \log_a y;$ $\log_a x^m = m \log_a x$	IN THE BOOKLET
Change of base	$\log_a x = \frac{\log_b x}{\log_b a}$	IN THE BOOKLET
Sum of an infinite geometric sequence Converges only when |r| < 1. Some booklet copies print the denominator as r minus 1, which is a misprint: use 1 minus r.	$S_\infty = \frac{u_1}{1 - r}, \quad \lvert r \rvert < 1$	IN THE BOOKLET
Binomial theorem The coefficient of the r+1th term is nCr.	$(a+b)^n = a^n + {}^nC_1\, a^{n-1}b + \cdots + {}^nC_r\, a^{n-r}b^r + \cdots + b^n$	IN THE BOOKLET
Binomial coefficient	${}^nC_r = \frac{n!}{r!\,(n-r)!}$	IN THE BOOKLET

Topic 1: Number and algebra, HL only

Combinations HL only. Order does not matter.	${}^nC_r = \frac{n!}{r!\,(n-r)!}$	IN THE BOOKLET
Permutations HL only. Order matters.	${}^nP_r = \frac{n!}{(n-r)!}$	IN THE BOOKLET
Complex number HL only. a is the real part, b the imaginary part.	$z = a + bi$	IN THE BOOKLET
Modulus-argument (polar) and exponential (Euler) form HL only. r is the modulus, θ the argument.	$z = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\,\mathrm{cis}\,\theta$	IN THE BOOKLET
De Moivre's theorem HL only. Use it for powers and roots of complex numbers.	$\big[r(\cos\theta + i\sin\theta)\big]^n = r^n(\cos n\theta + i\sin n\theta) = r^n e^{in\theta} = r^n\,\mathrm{cis}\,n\theta$	IN THE BOOKLET

Topic 2: Functions, SL and HL

Equations of a straight line Gradient-intercept, general and point-gradient forms.	$y = mx + c; \quad ax + by + d = 0; \quad y - y_1 = m(x - x_1)$	IN THE BOOKLET
Gradient formula Gradient through (x1, y1) and (x2, y2).	$m = \frac{y_2 - y_1}{x_2 - x_1}$	IN THE BOOKLET

Axis of symmetry of a quadratic The vertex lies on this line.	$f(x) = ax^2 + bx + c \Rightarrow x = -\frac{b}{2a}$	IN THE BOOKLET
Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a \neq 0$	IN THE BOOKLET
Discriminant Positive: two real roots. Zero: one repeated root. Negative: no real roots.	$\Delta = b^2 - 4ac$	IN THE BOOKLET
Exponential and logarithmic functions	$a^x = e^{x \ln a}; \quad \log_a a^x = x = a^{\log_a x}, \quad a, x > 0,\ a \neq 1$	IN THE BOOKLET

Topic 2: Functions, HL only

Sum and product of the roots of a polynomial equation HL only. For a polynomial of degree n.	$\sum_{r=0}^{n} a_r x^r = 0: \quad \text{sum} = \frac{-a_{n-1}}{a_n}; \quad \text{product} = \frac{(-1)^n a_0}{a_n}$	IN THE BOOKLET

Topic 3: Geometry and trigonometry, SL and HL

Distance between two points in three dimensions Points (x1, y1, z1) and (x2, y2, z2).	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$	IN THE BOOKLET
Midpoint of a line segment in three dimensions Endpoints (x1, y1, z1) and (x2, y2, z2).	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$	IN THE BOOKLET
Volume of a right pyramid A is the area of the base, h the height.	$V = \frac{1}{3}Ah$	IN THE BOOKLET
Volume of a right cone r is the radius, h the height.	$V = \frac{1}{3}\pi r^2 h$	IN THE BOOKLET
Curved surface area of a cone r is the radius, l the slant height.	$A = \pi r l$	IN THE BOOKLET
Volume of a sphere r is the radius.	$V = \frac{4}{3}\pi r^3$	IN THE BOOKLET
Surface area of a sphere r is the radius.	$A = 4\pi r^2$	IN THE BOOKLET
Sine rule Each side is opposite the angle with the same letter. Watch the ambiguous case (two sides and a non-included angle).	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	IN THE BOOKLET
Cosine rule Use the second form to find an angle from three sides.	$c^2 = a^2 + b^2 - 2ab\cos C; \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$	IN THE BOOKLET
Area of a triangle using sine C is the angle between sides a and b.	$A = \frac{1}{2}ab\sin C$	IN THE BOOKLET
Length of an arc r is the radius, θ the angle in radians.	$l = r\theta$	IN THE BOOKLET
Area of a sector r is the radius, θ the angle in radians.	$A = \frac{1}{2}r^2\theta$	IN THE BOOKLET
Identity for tan θ	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	IN THE BOOKLET
Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$	IN THE BOOKLET

Double angle identity for sine	$\sin 2\theta = 2\sin\theta\cos\theta$	IN THE BOOKLET
Double angle identity for cosine Pick the form that leaves only the function you need.	$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$	IN THE BOOKLET

Topic 3: Geometry and trigonometry, HL only

Reciprocal trigonometric identities HL only.	$\sec\theta = \frac{1}{\cos\theta};\quad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$	IN THE BOOKLET
Further Pythagorean identities HL only.	$1 + \tan^2\theta = \sec^2\theta;\quad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$	IN THE BOOKLET
Compound angle identity for sine HL only.	$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$	IN THE BOOKLET
Compound angle identity for cosine HL only. The sign flips between the two sides.	$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$	IN THE BOOKLET
Compound angle identity for tangent HL only.	$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$	IN THE BOOKLET
Double angle identity for tangent HL only.	$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$	IN THE BOOKLET
Magnitude of a vector HL only.	$\lvert\mathbf{v}\rvert = \sqrt{v_1^2 + v_2^2 + v_3^2},\quad \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$	IN THE BOOKLET
Scalar product (components) HL only.	$\mathbf{v}\cdot\mathbf{w} = v_1w_1 + v_2w_2 + v_3w_3,\quad \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix},\ \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$	IN THE BOOKLET
Scalar product (angle form) HL only. θ is the angle between v and w.	$\mathbf{v}\cdot\mathbf{w} = \lvert\mathbf{v}\rvert\lvert\mathbf{w}\rvert\cos\theta$	IN THE BOOKLET
Angle between two vectors HL only.	$\cos\theta = \frac{v_1w_1 + v_2w_2 + v_3w_3}{\lvert\mathbf{v}\rvert\lvert\mathbf{w}\rvert}$	IN THE BOOKLET
Vector equation of a line HL only. a is a point on the line, b the direction vector.	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$	IN THE BOOKLET
Parametric form of the equation of a line HL only.	$x = x_0 + \lambda l,\quad y = y_0 + \lambda m,\quad z = z_0 + \lambda n$	IN THE BOOKLET
Cartesian equations of a line HL only.	$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$	IN THE BOOKLET
Vector product (components) HL only.	$\mathbf{v}\times\mathbf{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$	IN THE BOOKLET
Vector product (magnitude) HL only. θ is the angle between v and w.	$\lvert\mathbf{v}\times\mathbf{w}\rvert = \lvert\mathbf{v}\rvert\lvert\mathbf{w}\rvert\sin\theta$	IN THE BOOKLET
Area of a parallelogram from vectors HL only. v and w form two adjacent sides.	$A = \lvert\mathbf{v}\times\mathbf{w}\rvert$	IN THE BOOKLET
Vector equation of a plane HL only.	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$	IN THE BOOKLET

Equation of a plane using the normal vector HL only. n is the normal vector.	$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$	IN THE BOOKLET
Cartesian equation of a plane HL only.	$ax + by + cz = d$	IN THE BOOKLET

Topic 4: Statistics and probability, SL and HL

Interquartile range	$\text{IQR} = Q_3 - Q_1$	IN THE BOOKLET
Mean of a set of data For grouped data use the midpoint of each class as x.	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}, \quad n = \sum_{i=1}^{k} f_i$	IN THE BOOKLET
Probability of an event A Equally likely outcomes: n(A) favourable out of n(U) in the sample space.	$P(A) = \frac{n(A)}{n(U)}$	IN THE BOOKLET
Complementary events	$P(A) + P(A') = 1$	IN THE BOOKLET
Combined events Subtract the overlap so it is not counted twice.	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	IN THE BOOKLET
Mutually exclusive events Only when A and B cannot happen together.	$P(A \cup B) = P(A) + P(B)$	IN THE BOOKLET
Conditional probability The probability of A given that B has happened.	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	IN THE BOOKLET
Independent events Only when A and B do not affect each other.	$P(A \cap B) = P(A)\,P(B)$	IN THE BOOKLET
Expected value of a discrete random variable	$E(X) = \sum x\,P(X = x)$	IN THE BOOKLET
Binomial distribution n trials, probability of success p on each.	$X \sim B(n, p)$	IN THE BOOKLET
Mean of a binomial distribution	$E(X) = np$	IN THE BOOKLET
Variance of a binomial distribution	$\text{Var}(X) = np(1-p)$	IN THE BOOKLET
Standardised normal variable Converts X ~ N(μ, sigma squared) to the standard normal Z.	$z = \frac{x - \mu}{\sigma}$	IN THE BOOKLET

Topic 4: Statistics and probability, HL only

Bayes' theorem (two events) HL only.	$P(B \mid A) = \frac{P(B)\,P(A \mid B)}{P(B)\,P(A \mid B) + P(B')\,P(A \mid B')}$	IN THE BOOKLET
Bayes' theorem (three events) HL only. B1, B2, B3 partition the sample space.	$P(B_i \mid A) = \frac{P(B_i)\,P(A \mid B_i)}{P(B_1)P(A \mid B_1) + P(B_2)P(A \mid B_2) + P(B_3)P(A \mid B_3)}$	IN THE BOOKLET
Variance of a data set HL only.	$\sigma^2 = \frac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n} = \frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$	IN THE BOOKLET
Standard deviation of a data set HL only.	$\sigma = \sqrt{\frac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}}$	IN THE BOOKLET

Linear transformation of a single random variable HL only. Adding b does not change the variance.	$E(aX+b) = aE(X)+b; \quad \operatorname{Var}(aX+b) = a^2\operatorname{Var}(X)$	IN THE BOOKLET
Expected value of a continuous random variable HL only.	$E(X) = \mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$	IN THE BOOKLET
Variance HL only.	$\operatorname{Var}(X) = E(X-\mu)^2 = E(X^2) - \left[E(X)\right]^2$	IN THE BOOKLET
Variance of a discrete random variable HL only.	$\operatorname{Var}(X) = \sum (x-\mu)^2 P(X=x) = \sum x^2 P(X=x) - \mu^2$	IN THE BOOKLET
Variance of a continuous random variable HL only.	$\operatorname{Var}(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$	IN THE BOOKLET

Topic 5: Calculus, SL and HL

Derivative of x^n	$f(x) = x^n \;\Rightarrow\; f'(x) = nx^{n-1}$	IN THE BOOKLET
Integral of x^n	$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	IN THE BOOKLET
Area between a curve and the x-axis Where f(x) > 0 between a and b.	$A = \int_a^b y\, dx$	IN THE BOOKLET
Derivative of sin x x in radians.	$f(x) = \sin x \;\Rightarrow\; f'(x) = \cos x$	IN THE BOOKLET
Derivative of cos x x in radians.	$f(x) = \cos x \;\Rightarrow\; f'(x) = -\sin x$	IN THE BOOKLET
Derivative of e^x	$f(x) = e^x \;\Rightarrow\; f'(x) = e^x$	IN THE BOOKLET
Derivative of ln x	$f(x) = \ln x \;\Rightarrow\; f'(x) = \frac{1}{x}$	IN THE BOOKLET
Chain rule	$y = g(u),\ u = f(x) \;\Rightarrow\; \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	IN THE BOOKLET
Product rule	$y = uv \;\Rightarrow\; \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$	IN THE BOOKLET
Quotient rule	$y = \frac{u}{v} \;\Rightarrow\; \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	IN THE BOOKLET
Acceleration v is velocity, s displacement, t time.	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	IN THE BOOKLET
Distance travelled from t1 to t2 The modulus counts movement in both directions.	$\text{distance} = \int_{t_1}^{t_2} \lvert v(t)\rvert\, dt$	IN THE BOOKLET
Displacement from t1 to t2 Signed: backwards movement is negative.	$\text{displacement} = \int_{t_1}^{t_2} v(t)\, dt$	IN THE BOOKLET
Standard integral of 1/x	$\int \frac{1}{x}\, dx = \ln\lvert x\rvert + C$	IN THE BOOKLET
Standard integral of sin x	$\int \sin x\, dx = -\cos x + C$	IN THE BOOKLET

Standard integral of cos x	$\int \cos x \, dx = \sin x + C$	IN THE BOOKLET
Standard integral of e^x	$\int e^x \, dx = e^x + C$	IN THE BOOKLET
Area of the region enclosed by a curve and the x-axis The modulus keeps area above zero when the curve dips below the axis.	$A = \int_a^b \lvert y \rvert \, dx$	IN THE BOOKLET

Topic 5: Calculus, HL only

Derivative from first principles HL only.	$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$	IN THE BOOKLET
Derivative of tan x HL only.	$f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$	IN THE BOOKLET
Derivative of sec x HL only.	$f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$	IN THE BOOKLET
Derivative of cosec x HL only.	$f(x) = \operatorname{cosec} x \Rightarrow f'(x) = -\operatorname{cosec} x \cot x$	IN THE BOOKLET
Derivative of cot x HL only.	$f(x) = \cot x \Rightarrow f'(x) = -\operatorname{cosec}^2 x$	IN THE BOOKLET
Derivative of a^x HL only.	$f(x) = a^x \Rightarrow f'(x) = a^x (\ln a)$	IN THE BOOKLET
Derivative of log base a of x HL only.	$f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \ln a}$	IN THE BOOKLET
Derivative of arcsin x HL only.	$f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1 - x^2}}$	IN THE BOOKLET
Derivative of arccos x HL only.	$f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1 - x^2}}$	IN THE BOOKLET
Derivative of arctan x HL only.	$f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1 + x^2}$	IN THE BOOKLET
Standard integral of a^x HL only.	$\int a^x \, dx = \frac{1}{\ln a} a^x + C$	IN THE BOOKLET
Standard integral giving arctan HL only.	$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \arctan \left(\frac{x}{a} \right) + C$	IN THE BOOKLET
Standard integral giving arcsin HL only.	$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \arcsin \left(\frac{x}{a} \right) + C, \quad \lvert x \rvert < a$	IN THE BOOKLET
Integration by parts HL only.	$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx \quad \text{or} \quad \int u \, dv = uv - \int v \, du$	IN THE BOOKLET
Area of the region enclosed by a curve and the y-axis HL only.	$A = \int_a^b \lvert x \rvert \, dy$	IN THE BOOKLET
Volume of revolution about the x or y axis HL only. Use the first for rotation about the x-axis, the second for the y-axis.	$V = \int_a^b \pi y^2 \, dx \quad \text{or} \quad V = \int_a^b \pi x^2 \, dy$	IN THE BOOKLET
Euler's method HL only. h is a constant step length.	$y_{n+1} = y_n + h \times f(x_n, y_n); \quad x_{n+1} = x_n + h$	IN THE BOOKLET

Integrating factor for y' + P(x)y = Q(x) HL only.	$e^{\int P(x)\,dx}$	IN THE BOOKLET
Maclaurin series HL only.	$f(x) = f(0) + x\,f'(0) + \frac{x^2}{2!}f''(0) + \dots$	IN THE BOOKLET
Maclaurin series for e^x HL only.	$e^x = 1 + x + \frac{x^2}{2!} + \dots$	IN THE BOOKLET
Maclaurin series for ln(1 + x) HL only.	$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$	IN THE BOOKLET
Maclaurin series for sin x HL only.	$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$	IN THE BOOKLET
Maclaurin series for cos x HL only.	$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$	IN THE BOOKLET
Maclaurin series for arctan x HL only.	$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$	IN THE BOOKLET

Learn these: not in the booklet

Laws of exponents

$a^m \times a^n = a^{m+n}$; $\frac{a^m}{a^n} = a^{m-n}$; $(a^m)^n = a^{mn}$; $a^{-n} = \frac{1}{a^n}$; $a^{1/n} = \sqrt[n]{a}$

Special logarithm values

$\log_a 1 = 0$; $\log_a a = 1$; $\ln e^x = x$

Parallel and perpendicular lines

$m_1 = m_2$ (parallel); $m_1 m_2 = -1$ (perpendicular)

Other forms of a quadratic

$f(x) = a(x-p)(x-q)$ gives roots p, q; $f(x) = a(x-h)^2 + k$ gives vertex (h, k)

Percentage change

$$\frac{\text{new} - \text{old}}{\text{old}} \times 100\%$$

Right-angled trigonometry

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$; $\cos\theta = \frac{\text{adj}}{\text{hyp}}$; $\tan\theta = \frac{\text{opp}}{\text{adj}}$; $a^2 + b^2 = c^2$

Exact trigonometric values

$\sin 30^\circ = \frac{1}{2}$; $\sin 45^\circ = \frac{\sqrt{2}}{2}$; $\sin 60^\circ = \frac{\sqrt{3}}{2}$; $\tan 30^\circ = \frac{\sqrt{3}}{3}$; $\tan 45^\circ = 1$; $\tan 60^\circ = \sqrt{3}$

Radians and degrees

π rad $= 180^\circ$

Unit circle symmetries

$\sin(\pi - \theta) = \sin\theta$; $\cos(\pi - \theta) = -\cos\theta$; $\sin(\frac{\pi}{2} - \theta) = \cos\theta$; $\cos(\frac{\pi}{2} - \theta) = \sin\theta$; $\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = \cos\theta$

Sinusoidal model

$f(x) = a\sin\big(b(x-c)\big) + d$; amplitude $|a|$; period $\frac{2\pi}{b}$; midline $y = d$

Rearranged double angle identities

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

Outliers

$x < Q_1 - 1.5 \times \text{IQR}$ or $x > Q_3 + 1.5 \times \text{IQR}$

Variance and standard deviation for SL (printed for HL only)

$\sigma^2 = \frac{\sum f(x-\mu)^2}{n}$; $\sigma = \sqrt{\sigma^2}$

Binomial probability

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$$

Tangent and normal at x = a

tangent $y - f(a) = f'(a)(x-a)$; normal gradient $= -\frac{1}{f'(a)}$

Stationary points and curvature

$f'(x) = 0$ at a stationary point; $f''(x) > 0$ minimum; $f''(x) < 0$ maximum; $f''(x) = 0$ with a sign change: point of inflection

Velocity

$$v = \frac{ds}{dt}$$

Reverse chain rule

$$\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$$

Area between two curves

$$A = \int_a^b \big(f(x) - g(x)\big)\,dx \text{ where } f(x) \geq g(x)$$

Modulus and conjugate of a complex number (HL)

$|z| = \sqrt{a^2+b^2}$; $z^* = a - bi$; $zz^* = |z|^2$

Factor and remainder theorems (HL)

$f(a) = 0 \Rightarrow (x - a)$ is a factor; remainder on dividing by $(x - a)$ is $f(a)$

Vector basics (HL)

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$; $\mathbf{v} \cdot \mathbf{w} = 0 \Rightarrow$ perpendicular; $\mathbf{v} = k\mathbf{w} \Rightarrow$ parallel

Integration by substitution (HL)

$$\int f\big(g(x)\big)g'(x)\,dx = \int f(u)\,du$$

L'Hopital's rule (HL)

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \text{ for } \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

Separable differential equations (HL)

$$\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{1}{g(y)}\,dy = \int f(x)\,dx$$

Homogeneous differential equations (HL)

substitute $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Using the booklet in the exam

- Practise with the booklet open all year, so you know which topic page holds each formula and can find it in seconds.
- Check the HL only rows before relying on them: AA SL students do not get Bayes' theorem, variance formulae or the extra trig identities.
- Write the infinite geometric sum as u1 divided by (1 minus r), and check |r| < 1 first. Some booklet copies misprint the denominator as r minus 1.
- Radian mode matters: the arc length, sector area and calculus formulae for sin and cos all assume angles in radians.
- For areas and distance travelled, remember the modulus signs in the printed integrals: split the integral at each zero crossing.

Based on: IB Mathematics: analysis and approaches (SL and HL), Mathematics: analysis and approaches formula booklet, version 1.0. Statistical tables are calculated by CuratedPrep to the same layout. Updated 31 October 2026. Always practise with the version of the official booklet your exam uses. More free sheets and topic worksheets at curatedprep.com.