FORMULA SHEET

IB maths formula booklet guide: AA and AI, SL and HL

The booklet prints formulas for all five topics, but AA and AI differ and SL prints less than HL. Check each row's course tag, then learn the recall list.

IN THE BOOKLET printed in the official booklet or sheet in the exam LEARN IT students must recall it

Number and algebra

Sequences, finance, logarithms, binomial, complex numbers and matrices. AI SL prints only sequences, compound interest, the exponent and logarithm link, and percentage error.

nth term of an arithmetic sequence AA and AI, SL and HL. u_1 is the first term, d the common difference.	$u_n = u_1 + (n-1)d$	IN THE BOOKLET
Sum of n terms of an arithmetic sequence AA and AI, SL and HL. Use the second form when you know the last term.	$S_n = \frac{n}{2}\left(2u_1 + (n-1)d\right) = \frac{n}{2}(u_1 + u_n)$	IN THE BOOKLET
nth term of a geometric sequence AA and AI, SL and HL. r is the common ratio.	$u_n = u_1 r^{\,n-1}$	IN THE BOOKLET
Sum of n terms of a finite geometric sequence AA and AI, SL and HL. Pick the form that keeps the denominator positive.	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$	IN THE BOOKLET
Sum to infinity of a geometric sequence AA SL and HL; AI HL only (not printed for AI SL). Version 1.0 of the AA booklet shows the denominator as r minus 1, which is a misprint: use 1 minus r.	$S_\infty = \frac{u_1}{1 - r}, \quad \lvert r\rvert < 1$	IN THE BOOKLET
Compound interest AA and AI, SL and HL. FV future value, PV present value, n years, k compounding periods per year, r% nominal annual rate.	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$	IN THE BOOKLET
Link between exponents and logarithms AA and AI, SL and HL.	$a^x = b \iff x = \log_a b, \quad a > 0,\ b > 0,\ a \neq 1$	IN THE BOOKLET
Laws of logarithms AA SL and HL; AI HL only (not printed for AI SL). Needs a, x, y > 0.	$\log_a xy = \log_a x + \log_a y; \quad \log_a \frac{x}{y} = \log_a x - \log_a y; \quad \log_a x^m = m \log_a x$	IN THE BOOKLET
Change of base AA SL and HL only; not printed for AI.	$\log_a x = \frac{\log_b x}{\log_b a}$	IN THE BOOKLET
Percentage error AI SL and HL only; not printed for AA. v_E exact value, v_A approximate value.	$\varepsilon = \left\lvert \frac{v_A - v_E}{v_E} \right\rvert \times 100\%$	IN THE BOOKLET
Binomial theorem AA SL and HL only; not printed for AI.	$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n$	IN THE BOOKLET

Binomial coefficient (combinations) AA SL and HL (printed with the binomial theorem, and again in the HL combinations entry); not printed for AI.	$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$	IN THE BOOKLET
Permutations AA HL only.	${}^{n}P_{r} = \frac{n!}{(n-r)!}$	IN THE BOOKLET
Complex number in Cartesian form AA HL and AI HL only.	$z = a + bi$	IN THE BOOKLET
Polar and Euler form of a complex number AA HL and AI HL only. r is the modulus, θ the argument.	$z = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\,\mathrm{cis}\,\theta$	IN THE BOOKLET
De Moivre's theorem AA HL only.	$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta) = r^{n}e^{in\theta}$	IN THE BOOKLET
Determinant of a 2 by 2 matrix AI HL only.	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det A = ad - bc$	IN THE BOOKLET
Inverse of a 2 by 2 matrix AI HL only.	$A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}, \quad ad \neq bc$	IN THE BOOKLET
Power of a matrix by diagonalisation AI HL only. P holds the eigenvectors, D is the diagonal matrix of eigenvalues.	$M^{n} = PD^{n}P^{-1}$	IN THE BOOKLET

Functions

Lines, quadratics, exponential functions and polynomial roots. AI prints very little here.

Equations of a straight line AA and AI, SL and HL.	$y = mx + c; \quad ax + by + d = 0; \quad y - y_1 = m(x - x_1)$	IN THE BOOKLET
Gradient from two points AA and AI, SL and HL.	$m = \frac{y_2 - y_1}{x_2 - x_1}$	IN THE BOOKLET
Axis of symmetry of a quadratic AA and AI, SL and HL. The vertex lies on this line.	$f(x) = ax^2 + bx + c \Rightarrow x = -\frac{b}{2a}$	IN THE BOOKLET
Quadratic formula AA SL and HL; AI HL only (prior learning list); not printed for AI SL.	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a \neq 0$	IN THE BOOKLET
Discriminant AA SL and HL; AI HL only. Positive: two real roots; zero: one repeated root; negative: no real roots.	$\Delta = b^2 - 4ac$	IN THE BOOKLET
Exponential and logarithmic functions AA SL and HL only.	$a^{x} = e^{x\ln a}; \quad \log_a a^{x} = x = a^{\log_a x}, \;\; a, x > 0, \; a \neq 1$	IN THE BOOKLET
Sum and product of roots of a polynomial AA HL only.	$\sum_{r=0}^{n} a_r x^{r} = 0: \;\; \text{sum} = \frac{-a_{n-1}}{a_n}, \;\; \text{product} = \frac{(-1)^{n}a_0}{a_n}$	IN THE BOOKLET
Logistic function AI HL only. L is the limiting value.	$f(x) = \frac{L}{1 + Ce^{-kx}}, \quad L, k, C > 0$	IN THE BOOKLET

Geometry and trigonometry

Includes the prior learning list of areas and volumes, which both booklets print for all four courses.

Area of a parallelogram Prior learning list. AA and AI, SL and HL.	$A = bh$	IN THE BOOKLET
Area of a triangle Prior learning list. AA and AI, SL and HL.	$A = \frac{1}{2}bh$	IN THE BOOKLET
Area of a trapezoid Prior learning list. AA and AI, SL and HL. a and b are the parallel sides.	$A = \frac{1}{2}(a+b)h$	IN THE BOOKLET
Area and circumference of a circle Prior learning list. AA and AI, SL and HL.	$A = \pi r^2; \quad C = 2\pi r$	IN THE BOOKLET
Volume of a cuboid, cylinder and prism Prior learning list. AA and AI, SL and HL. A is the cross-sectional area.	$V = lwh; \quad V = \pi r^2 h; \quad V = Ah$	IN THE BOOKLET
Curved surface area of a cylinder Prior learning list. AA and AI, SL and HL.	$A = 2\pi rh$	IN THE BOOKLET
Distance and midpoint in two dimensions Prior learning list. AA and AI, SL and HL.	$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}; \quad M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$	IN THE BOOKLET
Distance and midpoint in three dimensions AA and AI, SL and HL.	$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}; \quad M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$	IN THE BOOKLET
Volume of a right pyramid AA and AI, SL and HL. A is the base area.	$V = \frac{1}{3}Ah$	IN THE BOOKLET
Volume and curved surface area of a right cone AA and AI, SL and HL. l is the slant height.	$V = \frac{1}{3}\pi r^2 h; \quad A = \pi rl$	IN THE BOOKLET
Volume and surface area of a sphere AA and AI, SL and HL.	$V = \frac{4}{3}\pi r^3; \quad A = 4\pi r^2$	IN THE BOOKLET
Sine rule AA and AI, SL and HL. Watch for the ambiguous case when given two sides and a non-included angle.	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	IN THE BOOKLET
Cosine rule AA and AI, SL and HL.	$c^2 = a^2 + b^2 - 2ab\cos C; \quad \cos C = \frac{a^2+b^2-c^2}{2ab}$	IN THE BOOKLET
Area of a triangle using sine AA and AI, SL and HL.	$A = \frac{1}{2}ab\sin C$	IN THE BOOKLET
Arc length and sector area (radians) AA SL and HL; AI HL only. θ in radians.	$l = r\theta; \quad A = \frac{1}{2}r^2\theta$	IN THE BOOKLET
Arc length and sector area (degrees) AI SL and HL only; not printed for AA. θ in degrees.	$l = \frac{\theta}{360} \times 2\pi r; \quad A = \frac{\theta}{360} \times \pi r^2$	IN THE BOOKLET
Tangent identity AA SL and HL; AI HL only.	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	IN THE BOOKLET
Pythagorean identity AA SL and HL; AI HL only.	$\cos^2\theta + \sin^2\theta = 1$	IN THE BOOKLET

Double angle identities for sine and cosine AA SL and HL only.	$\sin 2\theta = 2\sin\theta\cos\theta; \quad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$	IN THE BOOKLET
Reciprocal trigonometric identities AA HL only.	$\sec\theta = \frac{1}{\cos\theta}; \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$	IN THE BOOKLET
Further Pythagorean identities AA HL only.	$1 + \tan^2\theta = \sec^2\theta; \quad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$	IN THE BOOKLET
Compound angle identities AA HL only.	$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B; \ \cos(A \pm B) = \cos A\cos B \mp \sin A\sin B; \ \tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$	IN THE BOOKLET
Double angle identity for tangent AA HL only.	$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$	IN THE BOOKLET
Reflection matrix in the line y = (tan θ)x AI HL only.	$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$	IN THE BOOKLET
Stretch matrices AI HL only. k is the scale factor.	$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ (parallel to x-axis); $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$ (parallel to y-axis)	IN THE BOOKLET
Enlargement matrix AI HL only. Scale factor k, centre (0, 0).	$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$	IN THE BOOKLET
Rotation matrices about the origin AI HL only. θ > 0.	$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ (anticlockwise); $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ (clockwise)	IN THE BOOKLET
Magnitude of a vector AA HL and AI HL only.	$\lvert\mathbf{v}\rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$	IN THE BOOKLET
Scalar product and angle between vectors AA HL and AI HL only.	$\mathbf{v}\cdot\mathbf{w} = v_1w_1 + v_2w_2 + v_3w_3 = \lvert\mathbf{v}\rvert\lvert\mathbf{w}\rvert\cos\theta; \quad \cos\theta = \frac{v_1w_1 + v_2w_2 + v_3w_3}{\lvert\mathbf{v}\rvert\lvert\mathbf{w}\rvert}$	IN THE BOOKLET
Vector equation of a line AA HL and AI HL only. a is a point on the line, b the direction.	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$	IN THE BOOKLET
Parametric form of a line AA HL and AI HL only.	$x = x_0 + \lambda l, \quad y = y_0 + \lambda m, \quad z = z_0 + \lambda n$	IN THE BOOKLET
Cartesian equations of a line AA HL only.	$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$	IN THE BOOKLET
Vector product and area of a parallelogram AA HL and AI HL only. The area is for the parallelogram with v and w as adjacent sides.	$\mathbf{v}\times\mathbf{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}; \ \lvert\mathbf{v}\times\mathbf{w}\rvert = \lvert\mathbf{v}\rvert\lvert\mathbf{w}\rvert\sin\theta; \ A = \lvert\mathbf{v}\times\mathbf{w}\rvert$	IN THE BOOKLET
Vector equation of a plane AA HL only.	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$	IN THE BOOKLET
Equation of a plane using the normal vector AA HL only.	$\mathbf{r}\cdot\mathbf{n} = \mathbf{a}\cdot\mathbf{n}$	IN THE BOOKLET
Cartesian equation of a plane AA HL only.	$ax + by + cz = d$	IN THE BOOKLET

Statistics and probability

AI SL prints no standard deviation or normal formula, so use the calculator for those.

Interquartile range AA and AI, SL and HL.	$\text{IQR} = Q_3 - Q_1$	IN THE BOOKLET
Mean of grouped or frequency data AA and AI, SL and HL.	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}, \quad n = \sum_{i=1}^{k} f_i$	IN THE BOOKLET
Probability of an event and complement AA and AI, SL and HL.	$P(A) = \frac{n(A)}{n(U)}; \quad P(A) + P(A') = 1$	IN THE BOOKLET
Combined events AA and AI, SL and HL.	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	IN THE BOOKLET
Mutually exclusive events AA and AI, SL and HL. Only when A and B cannot both happen.	$P(A \cup B) = P(A) + P(B)$	IN THE BOOKLET
Conditional probability AA and AI, SL and HL.	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	IN THE BOOKLET
Independent events AA and AI, SL and HL.	$P(A \cap B) = P(A)\,P(B)$	IN THE BOOKLET
Expected value of a discrete random variable AA and AI, SL and HL.	$E(X) = \sum x\,P(X = x)$	IN THE BOOKLET
Binomial distribution: mean and variance AA and AI, SL and HL. The probability formula itself is not printed.	$X \sim B(n, p) :\ E(X) = np, \quad \text{Var}(X) = np(1-p)$	IN THE BOOKLET
Standardised normal variable AA SL and HL only; not printed for AI.	$z = \frac{x - \mu}{\sigma}$	IN THE BOOKLET
Bayes' theorem AA HL only. The booklet also prints the version with three events B_1, B_2, B_3.	$P(B \mid A) = \frac{P(B)P(A \mid B)}{P(B)P(A \mid B) + P(B')P(A \mid B')}$	IN THE BOOKLET
Variance and standard deviation of grouped data AA HL only.	$\sigma^2 = \frac{\sum f_i(x_i - \mu)^2}{n} = \frac{\sum f_i x_i^2}{n} - \mu^2; \quad \sigma = \sqrt{\frac{\sum f_i(x_i - \mu)^2}{n}}$	IN THE BOOKLET
Linear transformation of a random variable AA HL and AI HL only. The constant b does not change the variance.	$E(aX + b) = aE(X) + b; \quad \text{Var}(aX + b) = a^2\text{Var}(X)$	IN THE BOOKLET
Expected value and variance of a continuous random variable AA HL only.	$E(X) = \mu = \int_{-\infty}^{\infty} x f(x)\,dx; \quad \text{Var}(X) = E(X^2) - [E(X)]^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$	IN THE BOOKLET
Variance of a discrete random variable AA HL only.	$\text{Var}(X) = \sum (x - \mu)^2 P(X = x) = \sum x^2 P(X = x) - \mu^2$	IN THE BOOKLET
Linear combinations of independent random variables AI HL only. Variances always add, even when the variables are subtracted.	$E(a_1X_1 \pm \cdots \pm a_nX_n) = a_1E(X_1) \pm \cdots \pm a_nE(X_n); \quad \text{Var}(a_1X_1 \pm \cdots \pm a_nX_n) = a_1^2\text{Var}(X_1) + \cdots + a_n^2\text{Var}(X_n)$	IN THE BOOKLET

Unbiased estimate of population variance AI HL only.	$s_{n-1}^2 = \frac{n}{n-1} s_n^2$	IN THE BOOKLET
Poisson distribution: mean and variance AI HL only.	$X \sim \mathrm{Po}(m) : \ E(X) = m, \quad \mathrm{Var}(X) = m$	IN THE BOOKLET
Transition matrices AI HL only. s_0 is the initial state.	$T^n s_0 = s_n$	IN THE BOOKLET

Calculus

AA SL prints the core rules; AI SL prints only the power rule, the area under a curve and the trapezoidal rule.

Derivative of x to the power n AA and AI, SL and HL.	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$	IN THE BOOKLET
Integral of x to the power n AA and AI, SL and HL.	$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	IN THE BOOKLET
Area under a curve AA and AI, SL and HL. The curve must be above the x-axis.	$A = \int_a^b y \, dx, \quad f(x) > 0$	IN THE BOOKLET
Trapezoidal rule AI SL and HL only; not printed for AA.	$\int_a^b y \, dx \approx \frac{1}{2} h\big((y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\big), \quad h = \frac{b-a}{n}$	IN THE BOOKLET
Derivatives of sin x, cos x, e^x and ln x AA SL and HL; AI HL only.	$\sin x \to \cos x; \quad \cos x \to -\sin x; \quad e^x \to e^x; \quad \ln x \to \frac{1}{x}$	IN THE BOOKLET
Chain rule AA SL and HL; AI HL only.	$y = g(u), \ u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	IN THE BOOKLET
Product rule AA SL and HL; AI HL only.	$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$	IN THE BOOKLET
Quotient rule AA SL and HL; AI HL only.	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	IN THE BOOKLET
Derivative of tan x AI HL only. AA HL prints the same result as sec squared x in its HL derivatives list.	$\tan x \to \frac{1}{\cos^2 x}$	IN THE BOOKLET
Acceleration, distance travelled and displacement AA SL and HL; AI HL only. AI HL also prints a = v dv/ds. Distance uses the modulus of v, so split the integral where v changes sign.	$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}; \quad \text{distance} = \int_{t_1}^{t_2} \lvert v(t) \rvert \, dt; \quad \text{displacement} = \int_{t_1}^{t_2} v(t) \, dt$	IN THE BOOKLET
Standard integrals (1/x, sin x, cos x, e^x) AA SL and HL; AI HL only. AI HL also prints the integral of 1 over cos squared x, which is tan x plus C.	$\int \frac{1}{x} dx = \ln \lvert x \rvert + C; \ \int \sin x \, dx = -\cos x + C; \ \int \cos x \, dx = \sin x + C; \ \int e^x dx = e^x + C$	IN THE BOOKLET
Area between a curve and the y-axis AA HL and AI HL only.	$A = \int_a^b x \, dy$	IN THE BOOKLET
Volume of revolution AA HL and AI HL only.	$V = \int_a^b \pi y^2 \, dx \ \text{ or } \ V = \int_a^b \pi x^2 \, dy$	IN THE BOOKLET

Euler's method AA HL and AI HL only. h is the step length.	$y_{n+1} = y_n + h \times f(x_n, y_n); \quad x_{n+1} = x_n + h$	IN THE BOOKLET
Euler's method for coupled systems AI HL only.	$x_{n+1} = x_n + hf_1(x_n, y_n, t_n);\ y_{n+1} = y_n + hf_2(x_n, y_n, t_n);\ t_{n+1} = t_n + h$	IN THE BOOKLET
Exact solution of coupled linear differential equations AI HL only. λ are eigenvalues, p are eigenvectors.	$\mathbf{x} = Ae^{\lambda_1 t}\mathbf{p}_1 + Be^{\lambda_2 t}\mathbf{p}_2$	IN THE BOOKLET
Derivative from first principles AA HL only.	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	IN THE BOOKLET
Derivatives of tan, sec, cosec and cot AA HL only.	$\tan x \to \sec^2 x;\ \sec x \to \sec x \tan x;\ \operatorname{cosec} x \to -\operatorname{cosec} x \cot x;\ \cot x \to -\operatorname{cosec}^2 x$	IN THE BOOKLET
Derivatives of a^x, log_a x and inverse trigonometric functions AA HL only.	$a^x \to a^x \ln a;\ \log_a x \to \frac{1}{x \ln a};\ \arcsin x \to \frac{1}{\sqrt{1-x^2}};\ \arccos x \to \frac{-1}{\sqrt{1-x^2}};\ \arctan x \to \frac{1}{1+x^2}$	IN THE BOOKLET
Further standard integrals AA HL only.	$\int a^x dx = \frac{1}{\ln a}a^x + C;\ \int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C;\ \int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C,\ \lvert x\rvert < a$	IN THE BOOKLET
Integration by parts AA HL only.	$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx \ \text{ or } \ \int u\,dv = uv - \int v\,du$	IN THE BOOKLET
Integrating factor AA HL only.	For $y' + P(x)y = Q(x)$: $e^{\int P(x)\,dx}$	IN THE BOOKLET
Maclaurin series AA HL only.	$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \ldots$	IN THE BOOKLET
Maclaurin series of standard functions AA HL only.	$e^x = 1 + x + \frac{x^2}{2!} + \ldots;\ \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots;\ \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots;\ \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots;\ \arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots$	IN THE BOOKLET

Learn these: not in the booklet

Laws of exponents (AA and AI)

$a^m \times a^n = a^{m+n};\ \frac{a^m}{a^n} = a^{m-n};\ (a^m)^n = a^{mn};\ a^{-n} = \frac{1}{a^n};\ a^{1/n} = \sqrt[n]{a}$

Special logarithm values (AA, and AI HL)

$\log_a 1 = 0;\ \log_a a = 1$

Parallel and perpendicular gradients (AA and AI)

$m_1 = m_2$ (parallel); $m_1 m_2 = -1$ (perpendicular)

Other forms of a quadratic (AA and AI)

$f(x) = a(x-p)(x-q)$ gives roots p, q; $f(x) = a(x-h)^2 + k$ gives vertex (h, k)

Percentage change (AA and AI)

$\frac{\text{new} - \text{old}}{\text{old}} \times 100\%$

Sinusoidal model (AA and AI)

$f(x) = a\sin\big(b(x-c)\big) + d$; amplitude $|a|$; period $\frac{2\pi}{b}$ or $\frac{360^\circ}{b}$; midline $y = d$

Right-angled trigonometry (AA and AI)

$\sin\theta = \frac{\text{opp}}{\text{hyp}};\ \cos\theta = \frac{\text{adj}}{\text{hyp}};\ \tan\theta = \frac{\text{opp}}{\text{adj}};\ a^2 + b^2 = c^2$

Exact values (AA)

$\sin 30^\circ = \frac{1}{2};\ \sin 45^\circ = \frac{\sqrt{2}}{2};\ \sin 60^\circ = \frac{\sqrt{3}}{2};\ \tan 30^\circ = \frac{\sqrt{3}}{3};\ \tan 45^\circ = 1;\ \tan 60^\circ = \sqrt{3}$

Radians and degrees (AA and AI HL)

π rad $= 180^\circ$

Trigonometric symmetries (AA)

$\sin(\pi-\theta)=\sin\theta;\ \cos(\pi-\theta)=-\cos\theta;\ \sin(\frac{\pi}{2}-\theta)=\cos\theta;\ \cos(\frac{\pi}{2}-\theta)=\sin\theta;\ \sin(-\theta)=-\sin\theta;\ \cos(-\theta)=\cos\theta$

Modulus and conjugate of a complex number (AA HL and AI HL)

$|z|=\sqrt{a^2+b^2};\ z^*=a-bi;\ zz^*=|z|^2$

Factor and remainder theorems (AA HL)

$f(a)=0\Rightarrow(x-a)$ is a factor; remainder on dividing by $(x-a)$ is $f(a)$

Vector basics (AA HL and AI HL)

$\overrightarrow{AB}=\mathbf{b}-\mathbf{a};\ \mathbf{v}\cdot\mathbf{w}=0\Rightarrow$ perpendicular; area of triangle $=\frac{1}{2}|\mathbf{v}\times\mathbf{w}|$

Outliers (AA and AI)

$x<Q_1-1.5\times\text{IQR}$ or $x>Q_3+1.5\times\text{IQR}$

Binomial probability (AA and AI)

$$P(X=r)=\binom{n}{r}p^r(1-p)^{n-r}$$

Standardising for a normal distribution (AI, since only AA prints it)

$$z=\frac{x-\mu}{\sigma}$$

Chi-squared test (AI)

$\chi^2=\sum\frac{(f_o-f_e)^2}{f_e};\ f_e=\frac{\text{row total}\times\text{column total}}{\text{grand total}};$ degrees of freedom $=(r-1)(c-1)$

Poisson probability (AI HL)

$$P(X=r)=\frac{e^{-m}m^r}{r!}$$

Distribution of the sample mean (AI HL)

$E(\bar{X})=\mu;\ \text{Var}(\bar{X})=\frac{\sigma^2}{n}$

Tangent and normal at x = a (AA and AI)

tangent $y-f(a)=f'(a)(x-a);$ normal gradient $=-\frac{1}{f'(a)}$

Stationary points and curvature (AA and AI)

$f'(x)=0$ at a stationary point; $f''(x)>0$ minimum; $f''(x)<0$ maximum; $f''(x)=0$ with a sign change: point of inflection

Velocity (AA and AI HL)

$$v=\frac{ds}{dt}$$

Reverse chain rule (AA)

$$\int f(ax+b)\,dx=\frac{1}{a}F(ax+b)+C$$

Area between two curves (AA)

$$A=\int_a^b\big(f(x)-g(x)\big)\,dx \text{ where } f(x)\geq g(x)$$

Integration by substitution (AA HL)

$$\int f\big(g(x)\big)g'(x)\,dx=\int f(u)\,du$$

L'Hopital's rule (AA HL)

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)} \text{ for } \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

Separable differential equations (AA HL)

$$\frac{dy}{dx}=f(x)g(y)\Rightarrow\int\frac{1}{g(y)}dy=\int f(x)\,dx$$

Homogeneous differential equations (AA HL)

substitute $y=vx,\ \frac{dy}{dx}=v+x\frac{dv}{dx}$

Eigenvalues and eigenvectors (AI HL)

$M\mathbf{v}=\lambda\mathbf{v};\ \det(M-\lambda I)=0$

Steady state of a transition matrix (AI HL)

$T\mathbf{s}=\mathbf{s}$

Using the sheet in the exam

- Open the booklet in practice sessions, so you know where each topic sits and can find a row in seconds.
- Check the course tag before relying on a row: AI SL students do not get the quadratic formula, log laws or trig identities printed.
- Copy a printed formula carefully: the AA booklet version 1.0 shows the infinite geometric sum with r minus 1 on the bottom, but the correct denominator is 1 minus r.
- Use the booklet for the pattern, then check units: compound interest needs the rate as r%, so r = 5 means 5 per cent, not 0.05.
- For anything the booklet does not print, such as tangent and normal equations, write the rule down before substituting.

Checked against the official documents: IB Mathematics: analysis and approaches (SL and HL), Mathematics: analysis and approaches formula booklet, version 1.0; IB Mathematics: applications and interpretation (SL and HL), Mathematics: applications and interpretation formula booklet, version 1.0. Updated 31 October 2026. Boards update their booklets from time to time: always practise with the version your exam uses. More free sheets and topic worksheets at curatedprep.com.