

Proof

7 questions · 45 marks

Answer all questions. The marks available are shown against each part. Every question is taken unaltered from a past paper; the source is noted beneath it.

1. Chloe is attempting to write $\frac{2x^2 + x}{(x + 1)(x + 2)^2}$ as partial fractions, with constant numerators. 7 marks

Her incorrect attempt is shown below.

Step 1	$\frac{2x^2 + x}{(x + 1)(x + 2)^2} \equiv \frac{A}{x + 1} + \frac{B}{(x + 2)^2}$
Step 2	$2x^2 + x \equiv A(x + 2)^2 + B(x + 1)$
Step 3	$\text{Let } x = -1 \Rightarrow A = 1$ $\text{Let } x = -2 \Rightarrow B = -6$
Answer	$\frac{2x^2 + x}{(x + 1)(x + 2)^2} \equiv \frac{1}{x + 1} - \frac{6}{(x + 2)^2}$

- (a)(i) By using a counter example, show that the answer obtained by Chloe cannot be correct.

[2 marks]

- (a)(ii) Explain her mistake in Step 1.

[1 mark]

- (b) Write $\frac{2x^2 + x}{(x + 1)(x + 2)^2}$ as partial fractions, with constant numerators.

[4 marks]

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— AQA 7357/1 Maths · June 2020 · Paper 1, Question 9

2. Kai is proving that $n^3 - n$ is a multiple of 3 for all positive integer values of n .

6 marks

Kai begins a proof by exhaustion.

Step 1	$n^3 - n = n(n^2 - 1)$
Step 2	When $n = 3m$, where m is a non-negative integer $n^3 - n = 3m(9m^2 - 1)$ which is a multiple of 3
Step 3	When $n = 3m + 1$, $n^3 - n = (3m + 1)((3m + 1)^2 - 1)$
Step 4	$= (3m + 1)(9m^2)$ $= 3(3m + 1)(3m^2)$ which is a multiple of 3
Step 5	Therefore $n^3 - n$ is a multiple of 3 for all positive integer values of n

- (a) Explain the two mistakes that Kai has made after Step 3.

[2 marks]

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(b) Correct Kai's argument from Step 4 onwards.

[4 marks]

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— AQA 7357/2 Maths · 2021 · Paper 2, Question 8

3. **(a)** Expand and simplify $(a - b)^2$

[1 mark]

6 marks

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(b) Peter thinks that the sum of any rational number and its reciprocal is always greater than 2

Peter checks two examples:

$$\frac{2}{3} + \frac{3}{2} = 2.1\bar{6}$$

$$2 + \frac{1}{2} = 2.5$$

Use a counter example to show that Peter is **incorrect**.

[2 marks]

(c) Given that a and b are distinct positive numbers, use proof by contradiction to prove that

$$\frac{a}{b} + \frac{b}{a} > 2$$

[3 marks]

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AQA 7357/2 Maths · 2023 · Paper 2, Question 10

4.

(a)

It is given that

6 marks

$$P(x) = 4x^3 + 8x^2 + 11x + 4$$

Use the factor theorem to show that $(2x + 1)$ is a factor of $P(x)$

[2 marks]

(b)

Express $P(x)$ in the form

$$P(x) = (2x + 1)(ax^2 + bx + c)$$

where a , b and c are constants to be found.

[2 marks]

(c)

Given that n is a positive integer, use your answer to part (b) to explain why $4n^3 + 8n^2 + 11n + 4$ is **never** prime.

[2 marks]

— AQA 7357/1 Maths · June 2024 · Paper 1, Question 13

5. a and b are two positive irrational numbers.

7 marks

The sum of a and b is rational.

The product of a and b is rational.

Caroline is trying to prove $\frac{1}{a} + \frac{1}{b}$ is rational.

Here is her proof:

Step 1 $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$

Step 2 2 is rational and $a+b$ is non-zero and rational.

Step 3 Therefore $\frac{2}{a+b}$ is rational.

Step 4 Hence $\frac{1}{a} + \frac{1}{b}$ is rational.

(a)(i) Identify Caroline's mistake.

[1 mark]

(a)(ii) Write down a correct version of the proof.

[2 marks]

(b) Prove by contradiction that the difference of any rational number and any irrational number is irrational.

[4 marks]

$27^2 = 729$	$29^2 = 841$
$2 + 7 = 9$	$2 + 9 = 11$
Last digit 9	Last digit 1

Give a counter example to show that Asif's conclusion is **not** correct.

$27^2 = 729$	$24^2 = 576$
$7^2 = 49$	$4^2 = 16$
Last digit 9	Last digit 6

[1 mark]

(c)

Given Claire's method is correct, use proof by exhaustion to show that no square number has a last digit of 8

[2 marks]

— AQA 7357/2 Maths · 2022 · Paper 2, Question 6

7.

(a)

Given that

8 marks

$$P(x) = 125x^3 + 150x^2 + 55x + 6$$

use the factor theorem to prove that $(5x + 1)$ is a factor of $P(x)$.

[2 marks]

(b)

Factorise $P(x)$ completely.

[3 marks]

(c)

Hence, prove that $250n^3 + 300n^2 + 110n + 12$ is a multiple of 12 when n is a positive whole number.

[3 marks]

— AQA 7357/1 Maths · 2021 · Paper 1, Question 13