

Composite and inverse functions

11 questions · 122 marks

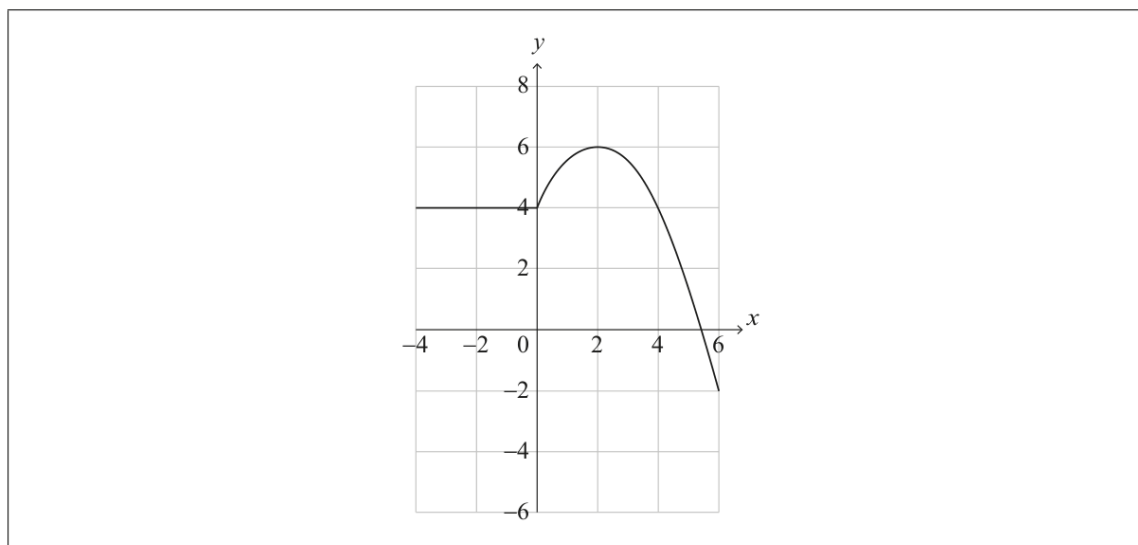
Answer all questions. The marks available are shown against each part. Every question is taken unaltered from a past paper; the source is noted beneath it. Some questions are taken from higher level papers, where they examine the same standard level content.

1.

[Maximum mark: 5]

5 marks

The graph of $y = f(x)$ for $-4 \leq x \leq 6$ is shown in the following diagram.



(a) Write down the value of

(i) $f(2)$;

(ii) $(f \circ f)(2)$.

[2]

(b) Let $g(x) = \frac{1}{2}f(x) + 1$ for $-4 \leq x \leq 6$. On the axes above, sketch the graph of g .

[3]

2.

[Maximum mark: 6]

6 marks

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 6x^2 - 12x + 1$ and $g(x) = -x + c$, where $c \in \mathbb{R}$.

(a) Find the range of f . [2]

(b) Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c . [4]

— IB 2221 – 7112 Maths AA · May 2021 Paper 2 TZ2 HL, Question 4

3.

[Maximum mark: 6]

6 marks

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 6x^2 - 12x + 1$ and $g(x) = -x + c$, where $c \in \mathbb{R}$.

(a) Find the range of f . [2]

(b) Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c . [4]

— IB 2221 – 7115 Maths AA · May 2021 Paper 2 TZ2 SL, Question 5

The function f is defined by $f(x) = \frac{7x+7}{2x-4}$ for $x \in \mathbb{R}$, $x \neq 2$.

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- This image shows a full page of primary-ruled paper. It features ten horizontal rows, each defined by two parallel dashed lines. Vertical solid lines are positioned on either side of the page to create margins. The entire sheet is enclosed within a thin black rectangular border.

5.

9 marks

[Maximum mark: 9]

The function f is defined by $f(x) = \frac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

(a) Write down the equation of

(i) the vertical asymptote of the graph of f ;

(ii) the horizontal asymptote of the graph of f .

[2]

(b) Find the coordinates where the graph of f crosses

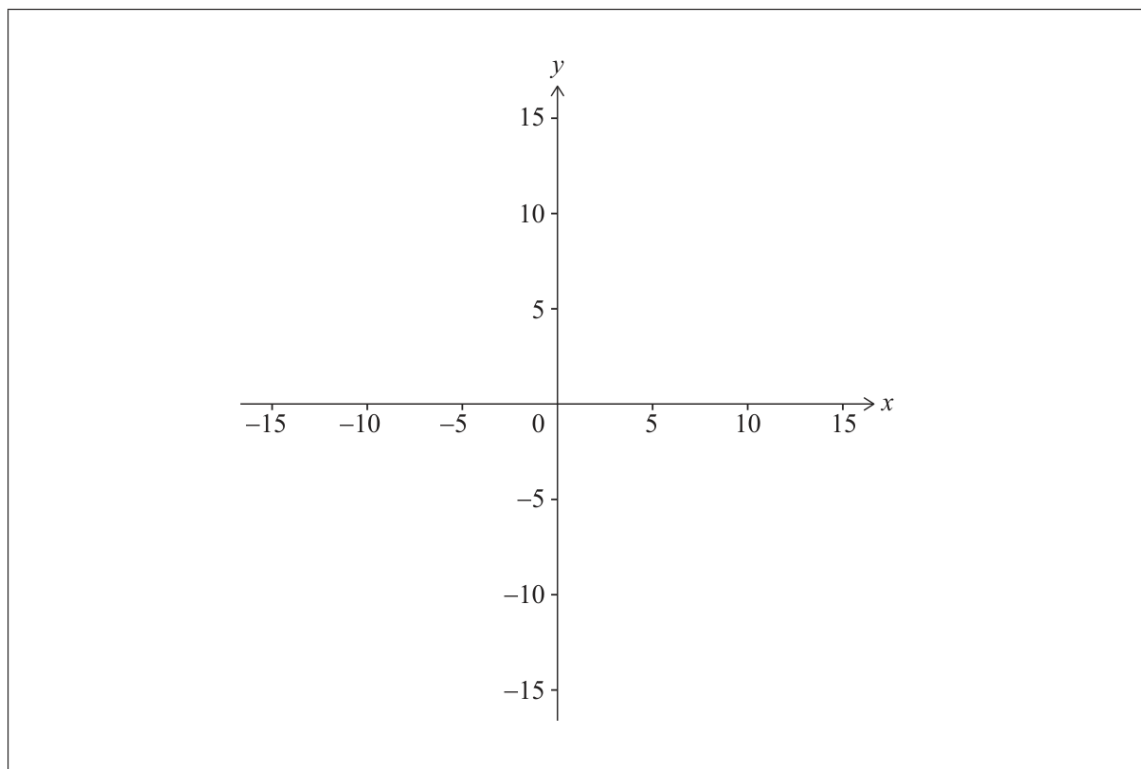
(i) the x -axis;

(ii) the y -axis.

[2]

(c) Sketch the graph of f on the axes below.

[1]



The function g is defined by $g(x) = \frac{ax+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$ and $a \in \mathbb{R}$.

(d) Given that $g(x) = g^{-1}(x)$, determine the value of a .

[4]

6.

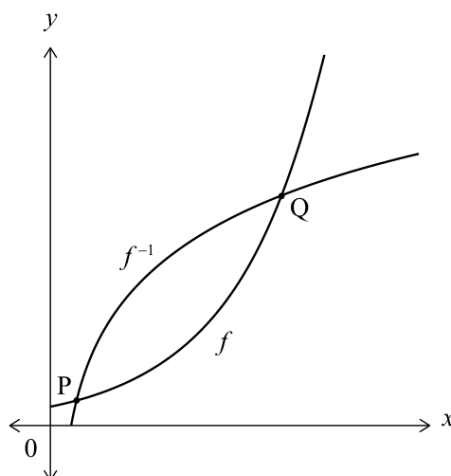
[Maximum mark: 12]

12 marks

Consider the function defined by $f(x) = \frac{3}{2}e^{x-2}$, $0 \leq x \leq 4$.

- (a) Show that the inverse function is given by $f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$. [3]

The graphs of f and f^{-1} intersect at two points P and Q, as shown on the following diagram.



- (b) Find PQ. [3]

The graph of f is reflected in the x -axis and then translated parallel to the y -axis by 5 units in the positive direction to give the graph of a function g .

- (c) Write down
- (i) an expression for $g(x)$;
 - (ii) the domain of g . [3]
- (d) Solve the equation $f(x) = g(x)$. Give your answer in the form $x = a + \ln b$, where $a, b \in \mathbb{Q}$. [3]

— IB 2224 – 7105 Maths AA · May 2024 Paper 2 TZ1 SL, Question 7

7.

14 marks

[Maximum mark: 14]

The function f is defined as $f(x) = \log_2(8x)$, where $x > 0$.

(a) Find the value of

(i) $f(2)$;

(ii) $f\left(\frac{1}{8}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(4x^2)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

— IB 8824 – 7104 Maths AA · November 2024 Paper 1 TZ1 SL, Question 8

8.

14 marks

[Maximum mark: 14]

The function f is defined as $f(x) = \log_2(4x)$, where $x > 0$.

(a) Find the value of

(i) $f(8)$;

(ii) $f\left(\frac{1}{4}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(16x^3)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

— IB 8824 – 7109 Maths AA · November 2024 Paper 1 TZ2 SL, Question 8

9.

15 marks

[Maximum mark: 15]

Consider the function $f(x) = \frac{2-2x}{x+2}$, where $x \in \mathbb{R}$, $x \neq -2$.

(a) Show that $f^{-1}(x) = f(x)$. [3]

The point $P\left(k, \frac{2-2k}{k+2}\right)$ is the point on the graph of $y = f(x)$ that is closest to the origin $(0, 0)$.

(b) (i) Find the value of k .

(ii) Hence, write down the coordinates of P . [4]

Consider the function $g(x) = \frac{2-3x}{cx+d}$, where $x \in \mathbb{R}$, $x \neq -\frac{d}{c}$, and $c, d \neq 0$.

The graph of $y = g(x)$ has a vertical asymptote and a horizontal asymptote.

(c) In terms of c and d , write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

It is given that $g^{-1}(x) = g(x)$.

(d) Find the value of d . [2]

Consider the case where $c = 1$.

(e) Sketch the graph of $y = \frac{1}{g(x)}$, showing the values of any intercepts with the axes and including any asymptotes, labelled with their equations. [4]

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10.

17 marks

[Maximum mark: 17]

The function f is defined by $f(x) = 4^x$, where $x \in \mathbb{R}$.

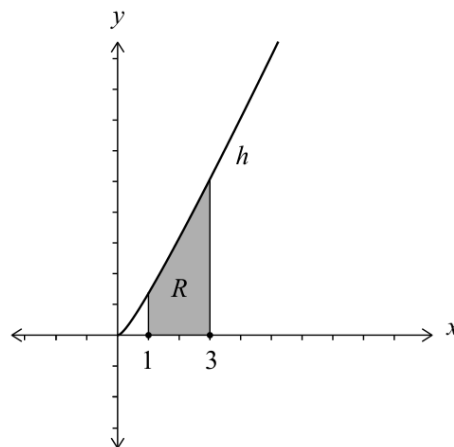
- (a) Find $f^{-1}(8)$. Express your answer in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$. [3]

The function g is defined by $g(x) = 1 + \log_2 x$, where $x \in \mathbb{R}^+$.

- (b) (i) Find an expression for $g^{-1}(x)$.
 (ii) Describe a sequence of transformations that transforms the graph of $y = g^{-1}(x)$ to the graph of $y = f(x)$. [4]
 (c) Show that $(f \circ g)(x) = 4x^2$. [3]

The function h is defined by $h(x) = \frac{4x^2}{2x+1}$, $x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of h . Let R be the region enclosed by the graph of h and the x -axis, between the lines $x = 1$ and $x = 3$.



- (d) (i) Show that $2x - 1 + \frac{1}{2x+1} = \frac{4x^2}{2x+1}$.
 (ii) Hence or otherwise, find the area of R , giving your answer in the form $p + q \ln r$, where $p, q, r \in \mathbb{Q}^+$. [7]

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11.

17 marks

[Maximum mark: 17]

The function f is defined by $f(x) = 4^x$, where $x \in \mathbb{R}$.

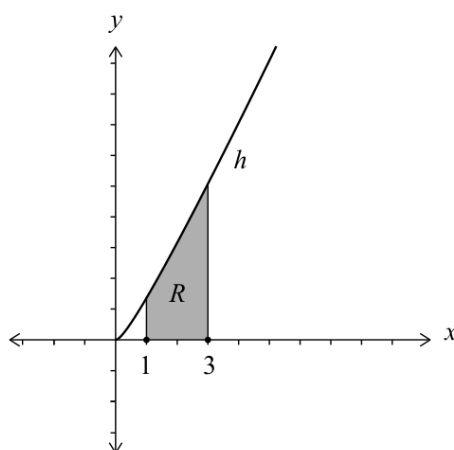
- (a) Find $f^{-1}(8)$. Express your answer in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$. [3]

The function g is defined by $g(x) = 1 + \log_2 x$, where $x \in \mathbb{R}^+$.

- (b) (i) Find an expression for $g^{-1}(x)$.
 (ii) Describe a sequence of transformations that transforms the graph of $y = g^{-1}(x)$ to the graph of $y = f(x)$. [4]
 (c) Show that $(f \circ g)(x) = 4x^2$. [3]

The function h is defined by $h(x) = \frac{4x^2}{2x+1}$, $x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of h . Let R be the region enclosed by the graph of h and the x -axis, between the lines $x = 1$ and $x = 3$.



- (d) (i) Show that $2x - 1 + \frac{1}{2x+1} = \frac{4x^2}{2x+1}$.
 (ii) Hence or otherwise, find the area of R , giving your answer in the form $p + q \ln r$, where $p, q, r \in \mathbb{Q}^+$. [7]

IB 2225 – 7111 Maths AA · May 2025 Paper 1 TZ2 HL, Question 10
