

Exponential and logarithmic functions

16 questions · 170 marks

Answer all questions. The marks available are shown against each part. Every question is taken unaltered from a past paper; the source is noted beneath it. Some questions are taken from higher level papers, where they examine the same standard level content.

1. [Maximum mark: 5] 5 marks

Consider the function $h(x) = \log_{10}(4x^2 - rx + r - 1)$, where $x \in \mathbb{R}$.

Find the possible values of r . [5]

IB 8824 – 7110 Maths AA · November 2024 Paper 2 TZ2 SL, Question 6

2.

6 marks

[Maximum mark: 6]

The loudness of a sound, L , measured in decibels, is related to its intensity, I units, by $L = 10 \log_{10}(I \times 10^{12})$.

Consider two sounds, S_1 and S_2 .

S_1 has an intensity of 10^{-6} units and a loudness of 60 decibels.

S_2 has an intensity that is twice that of S_1 .

(a) State the intensity of S_2 . [1]

(b) Determine the loudness of S_2 . [2]

The maximum loudness of thunder in a thunderstorm was measured to be 115 decibels.

(c) Find the corresponding intensity, I , of the thunder. [3]

— IB 2224 – 7107 Maths AA · May 2024 Paper 2 TZ2 HL, Question 3

3.

6 marks

[Maximum mark: 6]

The functions f and g are both defined for $-1 \leq x \leq 0$ by

$$f(x) = 1 - x^2$$

$$g(x) = e^{2x}.$$

The graphs of f and g intersect at $x = a$ and $x = b$, where $a < b$.

(a) Find the value of a and the value of b . [3]

(b) Find the area of the region enclosed by the graphs of f and g . [3]

— IB 2224 – 7107 Maths AA · May 2024 Paper 2 TZ2 HL, Question 1

[illegible]

4.

[Maximum mark: 6]

6 marks

Find the range of possible values of k such that $e^{2x} + \ln k = 3e^x$ has at least one real solution.

— IB 2223 – 7109 Maths AA · May 2023 Paper 1 TZ1 SL, Question 5

[illegible]

5.

6 marks

[Maximum mark: 6]

The loudness of a sound, L , measured in decibels, is related to its intensity, I units, by $L = 10 \log_{10}(I \times 10^{12})$.

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S_1 has an intensity of 10^{-6} units and a loudness of 60 decibels.

S_2 has an intensity that is twice that of S_1 .

(a) State the intensity of S_2 . [1]

(b) Determine the loudness of S_2 . [2]

The maximum loudness of thunder in a thunderstorm was measured to be 115 decibels.

(c) Find the corresponding intensity, I , of the thunder. [3]

— IB 2224 – 7110 Maths AA · May 2024 Paper 2 TZ2 SL, Question 4

6.

7 marks

[Maximum mark: 7]

All living plants contain an isotope of carbon called carbon-14. When a plant dies, the isotope decays so that the amount of carbon-14 present in the remains of the plant decreases. The time since the death of a plant can be determined by measuring the amount of carbon-14 still present in the remains.

The amount, A , of carbon-14 present in a plant t years after its death can be modelled by $A = A_0 e^{-kt}$ where $t \geq 0$ and A_0 , k are positive constants.

At the time of death, a plant is defined to have 100 units of carbon-14.

(a) Show that $A_0 = 100$. [1]

The time taken for half the original amount of carbon-14 to decay is known to be 5730 years.

(b) Show that $k = \frac{\ln 2}{5730}$. [3]

(c) Find, correct to the nearest 10 years, the time taken after the plant's death for 25% of the carbon-14 to decay. [3]

[illegible]

7 marks

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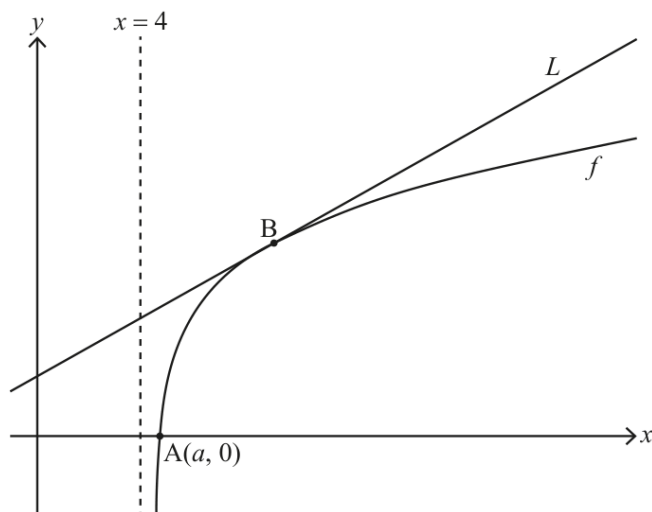
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Page 5

9 marks

The following diagram shows part of the graph of f which crosses the x -axis at point A, with coordinates $(a, 0)$. The line L is the tangent to the graph of f at the point B.



- (a) Find the exact value of α . [3]
- (b) Given that the gradient of L is $\frac{1}{3}$, find the x -coordinate of B. [6]

[illegible]

9.

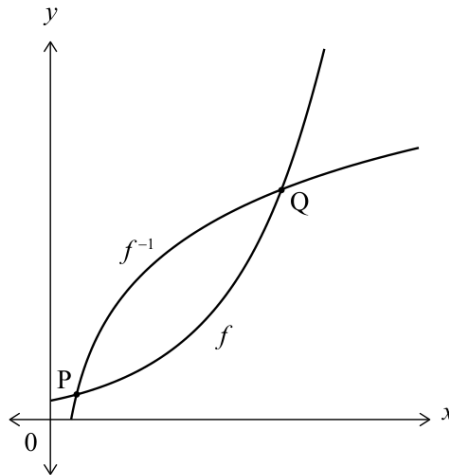
12 marks

[Maximum mark: 12]

Consider the function defined by $f(x) = \frac{3}{2}e^{x-2}$, $0 \leq x \leq 4$.

- (a) Show that the inverse function is given by $f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$. [3]

The graphs of f and f^{-1} intersect at two points P and Q, as shown on the following diagram.



- (b) Find PQ. [3]

The graph of f is reflected in the x -axis and then translated parallel to the y -axis by 5 units in the positive direction to give the graph of a function g .

- (c) Write down
- (i) an expression for $g(x)$;
 - (ii) the domain of g . [3]
- (d) Solve the equation $f(x) = g(x)$. Give your answer in the form $x = a + \ln b$, where $a, b \in \mathbb{Q}$. [3]

— IB 2224 – 7105 Maths AA · May 2024 Paper 2 TZ1 SL, Question 7

10.

14 marks

[Maximum mark: 14]

The function f is defined as $f(x) = \log_2(8x)$, where $x > 0$.

(a) Find the value of

(i) $f(2)$;

(ii) $f\left(\frac{1}{8}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(4x^2)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

— IB 8824 – 7104 Maths AA · November 2024 Paper 1 TZ1 SL, Question 8

11.

14 marks

[Maximum mark: 14]

The function f is defined as $f(x) = \log_2(4x)$, where $x > 0$.

(a) Find the value of

(i) $f(8)$;

(ii) $f\left(\frac{1}{4}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(16x^3)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

— IB 8824 – 7109 Maths AA · November 2024 Paper 1 TZ2 SL, Question 8

12.

[Maximum mark: 15]

15 marks

The following table shows the population of Canada t years after the year 2000.

t (years after 2000)	0	5	10	15	20
p (population in millions)	30.7	32.2	34.0	35.7	37.9

A student uses linear regression to model the population of Canada using these data. The student model is $p = at + b$.

- (a) (i) Write down the value of a and the value of b .
 (ii) Interpret, in context, the value of a . [3]

The student uses this model to predict the population of Canada in the year 2030, where $t = 30$, and calculates a population of approximately 41.3 million people.

- (b) Comment on the reliability of the student's prediction. [1]

A data scientist, Benoit, uses additional information to develop an exponential model for Canada's future population.

In this model, $B(t) = 33.5(1.005)^t$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

- (c) (i) Use Benoit's model to predict the population of Canada in the year 2100.
 (ii) Interpret, in context, the value 1.005 in Benoit's model. [3]

Another data scientist, Cecilia, develops a third model for the Canadian population.

In this model, $C(t) = \frac{62}{1 + e^{-0.02t}}$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

- (d) Use Cecilia's model to predict the population of Canada in the year 2100. [1]
- (e) Determine the year in which the difference between the predictions from Benoit's model and Cecilia's model is greatest. [3]
- (f) Find the value of
 - (i) $B'(75)$;
 - (ii) $C'(75)$. [2]
- (g) Compare and interpret, in context, the values of $B'(75)$ and $C'(75)$. [2]

— IB 8824 – 7110 Maths AA • November 2024 Paper 2 TZ2 SL, Question 8

[illegible]

13.

15 marks

[Maximum mark: 15]

The following table shows the population of Canada t years after the year 2000.

t (years after 2000)	0	5	10	15	20
p (population in millions)	30.6	32.3	34.1	35.6	38.0

A student uses linear regression to model the population of Canada using these data. The student model is $p = at + b$.

- (a) (i) Write down the value of a and the value of b .
 (ii) Interpret, in context, the value of a . [3]

The student uses this model to predict the population of Canada in the year 2030, where $t = 30$, and calculates a population of approximately 41.4 million people.

- (b) Comment on the reliability of the student's prediction. [1]

A data scientist, Benoit, uses additional information to develop an exponential model for Canada's future population.

In this model, $B(t) = 30.6(1.007)^t$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

- (c) (i) Use Benoit's model to predict the population of Canada in the year 2100.
 (ii) Interpret, in context, the value 1.007 in Benoit's model. [3]

Another data scientist, Cecilia, develops a third model for the Canadian population.

In this model, $C(t) = \frac{61}{1 + e^{-0.03t}}$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

- (d) Use Cecilia's model to predict the population of Canada in the year 2100. [1]
 (e) Determine the year in which the difference between the predictions from Benoit's model and Cecilia's model is greatest. [3]
 (f) Find the value of
 (i) $B'(40)$;
 (ii) $C'(40)$. [2]
 (g) Compare and interpret, in context, the values of $B'(40)$ and $C'(40)$. [2]

[illegible]

14.

[Maximum mark: 15]

15 marks

The following table shows the population of Canada t years after the year 2000.

t (years after 2000)	0	5	10	15	20
p (population in millions)	30.7	32.2	34.0	35.7	37.9

A student uses linear regression to model the population of Canada using these data. The student model is $p = at + b$.

- (a) (i) Write down the value of a and the value of b .
- (ii) Interpret, in context, the value of a . [3]

The student uses this model to predict the population of Canada in the year 2030, where $t = 30$, and calculates a population of approximately 41.3 million people.

- (b) Comment on the reliability of the student's prediction. [1]

A data scientist, Benoit, uses additional information to develop an exponential model for Canada's future population.

In this model, $B(t) = 33.5(1.005)^t$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

- (c) (i) Use Benoit's model to predict the population of Canada in the year 2100.
- (ii) Interpret, in context, the value 1.005 in Benoit's model. [3]

Another data scientist, Cecilia, develops a third model for the Canadian population.

In this model, $C(t) = \frac{62}{1 + e^{-0.02t}}$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

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- (f) Find the value of
 - (i) $B'(75)$;
 - (ii) $C'(75)$. [2]
- (g) Compare and interpret, in context, the values of $B'(75)$ and $C'(75)$. [2]

IB 8824 - 9701 Maths AA · November 2024 Paper 2 HL, Question 10

15.

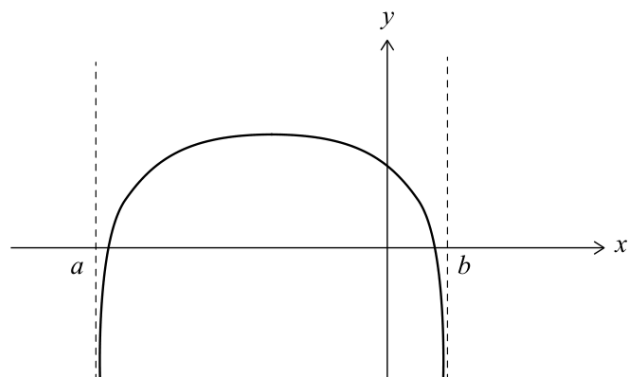
16 marks

[Maximum mark: 16]

(a) (i) Solve $5 - 4x - x^2 = 0$.

(ii) Hence, find the values of x such that $5 - 4x - x^2 > 0$. [4]

Consider the function $f(x) = \log_k(5 - 4x - x^2)$, where $a < x < b$ and $k > 1$. Part of the graph of f is shown in the following diagram.



The graph of f has vertical asymptotes at $x = a$ and $x = b$.

(b) Write down the value of

(i) a ;

(ii) b . [2]

(c) Find the exact values of x such that $f(x) = 0$. [4]

The graph of f has a maximum value of 2.

(d) Find the value of k . [6]

— IB 2225 – 7119 Maths AA · May 2025 Paper 1 TZ3 SL, Question 9

16.

17 marks

[Maximum mark: 17]

The function f is defined by $f(x) = 4^x$, where $x \in \mathbb{R}$.

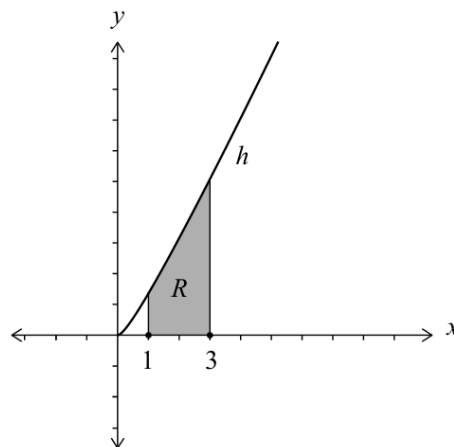
- (a) Find $f^{-1}(8)$. Express your answer in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$. [3]

The function g is defined by $g(x) = 1 + \log_2 x$, where $x \in \mathbb{R}^+$.

- (b) (i) Find an expression for $g^{-1}(x)$.
 (ii) Describe a sequence of transformations that transforms the graph of $y = g^{-1}(x)$ to the graph of $y = f(x)$. [4]
 (c) Show that $(f \circ g)(x) = 4x^2$. [3]

The function h is defined by $h(x) = \frac{4x^2}{2x+1}$, $x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of h . Let R be the region enclosed by the graph of h and the x -axis, between the lines $x = 1$ and $x = 3$.



- (d) (i) Show that $2x - 1 + \frac{1}{2x+1} = \frac{4x^2}{2x+1}$.
 (ii) Hence or otherwise, find the area of R , giving your answer in the form $p + q \ln r$, where $p, q, r \in \mathbb{Q}^+$. [7]

— IB 2225 – 7114 Maths AA · May 2025 Paper 1 TZ2 SL, Question 9
