

Quadratic functions and equations

20 questions · 173 marks

Answer all questions. The marks available are shown against each part. Every question is taken unaltered from a past paper; the source is noted beneath it. Some questions are taken from higher level papers, where they examine the same standard level content.

1. [Maximum mark: 5] 5 marks

(a) Solve $3m^2 + 5m - 2 = 0$. [3]

(b) Hence or otherwise, solve $3 \times 9^x + 5 \times 3^x - 2 = 0$. [2]

IB 2224 – 7109 Maths AA · May 2024 Paper 1 TZ2 SL, Question 3

2. [Maximum mark: 5] 5 marks

Consider the function $h(x) = \log_{10}(3x^2 - rx + r - 2)$, where $x \in \mathbb{R}$.

Find the possible values of r . [5]

IB 8824 – 7105 Maths AA · November 2024 Paper 2 TZ1 SL, Question 6

[illegible]

3.

[Maximum mark: 5]

5 marks

Solve $3 \times 9^x + 5 \times 3^x - 2 = 0$.

— IB 2224 – 7106 Maths AA · May 2024 Paper 1 TZ2 HL, Question 2

[illegible]

4.

[Maximum mark: 5]

5 marks

Consider the function $h(x) = \log_{10}(4x^2 - rx + r - 1)$, where $x \in \mathbb{R}$.

Find the possible values of r .

[5]

— IB 8824 – 9701 Maths AA · November 2024 Paper 2 HL, Question 5

5.

[Maximum mark: 5]

5 marks

Consider the function $h(x) = \log_{10}(4x^2 - rx + r - 1)$, where $x \in \mathbb{R}$.

Find the possible values of r .

[5]

— IB 8824 – 7110 Maths AA · November 2024 Paper 2 TZ2 SL, Question 6

6 marks

(b) Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c . [4]

— IB 2221 – 7115 Maths AA · May 2021 Paper 2 TZ2 SL, Question 5

[illegible]

7.

6 marks

— IB 2223 – 7109 Maths AA · May 2023 Paper 1 TZ1 SL, Question 5

[illegible]

8.

[Maximum mark: 6]

6 marks

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 6x^2 - 12x + 1$ and $g(x) = -x + c$, where $c \in \mathbb{R}$.

(a) Find the range of f . [2]

(b) Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c . [4]

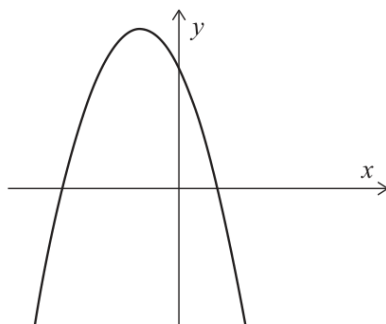
— IB 2221 – 7112 Maths AA · May 2021 Paper 2 TZ2 HL, Question 4

9.

[Maximum mark: 7]

7 marks

Consider the function $f(x) = -2(x - 1)(x + 3)$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f .



- (a) For the graph of f
- (i) find the x -coordinates of the x -intercepts;
 - (ii) find the coordinates of the vertex. [5]

The function f can be written in the form $f(x) = -2(x - h)^2 + k$.

- (b) Write down the value of h and the value of k . [2]

— IB 8821 – 7104 Maths AA · November 2021 Paper 1 SL, Question 1

[illegible]

10.

[Maximum mark: 7]

7 marks

Consider the curve $y = x^2 - x - 1$ and the line $y = mx - 3$, where $m \in \mathbb{R}$.

- (a) Show that the curve and the line meet when $x^2 - (m + 1)x + 2 = 0$. [2]
- (b) Hence, find the values of m when the line is tangent to the curve. [5]

— IB 2225 – 7119 Maths AA · May 2025 Paper 1 TZ3 SL, Question 5

[illegible]

11. [Maximum mark: 7] 7 marks

The quadratic equation $x^2 + kx + 15 - k = 0$ has two distinct real roots.

- (a) Find the possible values of k . [5]
- (b) Find the possible values of k in the case where the two distinct real roots are both positive or both negative. [2]

— IB 2225 – 7114 Maths AA · May 2025 Paper 1 TZ2 SL, Question 5

12. [Maximum mark: 7] 7 marks

The equation $3px^2 + 2px + 1 = p$ has two real, distinct roots.

- (a) Find the possible values for p . [5]
- (b) Consider the case when $p = 4$. The roots of the equation can be expressed in the form $x = \frac{a \pm \sqrt{13}}{6}$, where $a \in \mathbb{Z}$. Find the value of a . [2]

— IB 8821 – 7101 Maths AA · November 2021 Paper 1 HL, Question 7

13.

[Maximum mark: 7]

7 marks

Consider the curve $y = x^2 - x - 1$ and the line $y = mx - 3$, where $m \in \mathbb{R}$.

- (a) Show that the curve and the line meet when $x^2 - (m + 1)x + 2 = 0$. [2]
- (b) Hence, find the values of m when the line is tangent to the curve. [5]

— IB 2225 – 7116 Maths AA · May 2025 Paper 1 TZ3 HL, Question 4

14.

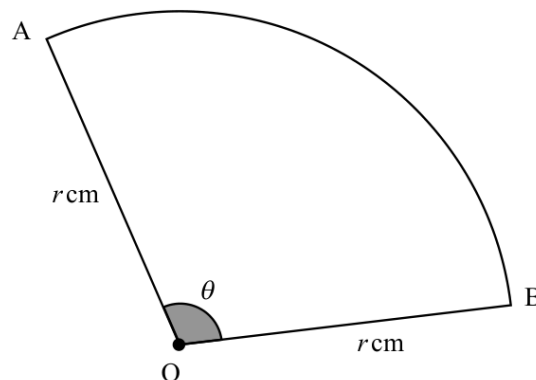
[Maximum mark: 8]

8 marks

Points A and B lie on the circumference of a circle of radius r cm with centre at O.

The sector OAB is shown on the following diagram. The angle $\widehat{AOB}$ is denoted as θ and is measured in radians.

diagram not to scale



The perimeter of the sector is 10 cm and the area of the sector is 6.25 cm^2 .

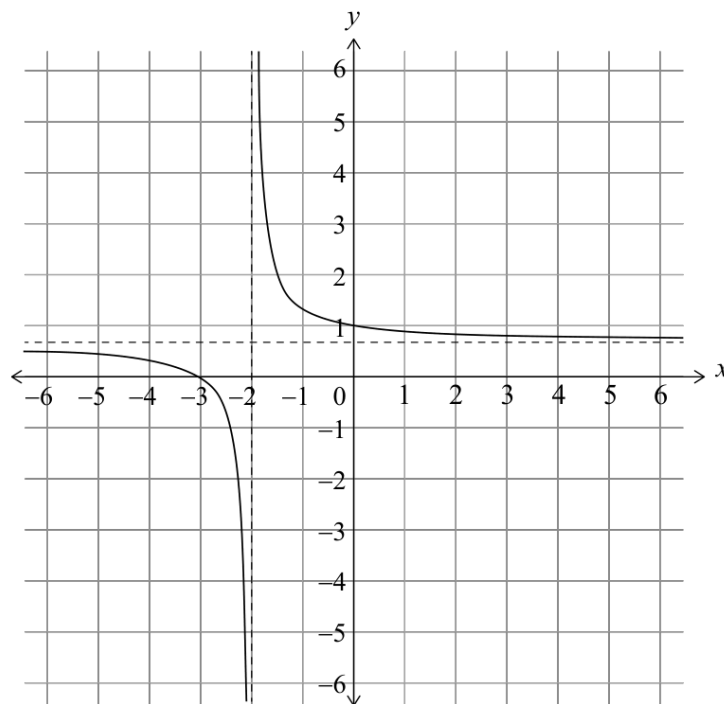
- (a) Show that $4r^2 - 20r + 25 = 0$. [4]
- (b) Hence, or otherwise, find the value of r and the value of θ . [4]

— IB 2224 – 7104 Maths AA · May 2024 Paper 1 TZ1 SL, Question 4

15. [Maximum mark: 8] 8 marks

A function f is defined by $f(x) = \frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R}, x \neq -2$.

The graph $y = f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

Consider $g(x) = mx + 1$, where $m \in \mathbb{R}, m \neq 0$.

- (b) (i) Write down the number of solutions to $f(x) = g(x)$ for $m > 0$.
- (ii) Determine the value of m such that $f(x) = g(x)$ has only one solution for x .
- (iii) Determine the range of values for m , where $f(x) = g(x)$ has two solutions for $x \geq 0$. [7]

[illegible]

14 marks

(d) Hence find the values of x where the graph of f is both negative and increasing. [3]

— IB 2221 - 7109 Maths AA · May 2021 Paper 1 TZ1 SL, Question 7

17.

15 marks

[Maximum mark: 15]

The function f is defined by $f(x) = 5(x + 1)(x + 3)$, where $x \in \mathbb{R}$.

- (a) Write $f(x)$ in the form $a(x - h)^2 + k$, where $a, h, k \in \mathbb{Z}$. [4]
- (b) Sketch the graph of $y = f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex. [4]
- (c) Solve the inequality $f(x) \leq 40$. [4]

The function g is defined by $g(x) = \ln x$, where $x \in \mathbb{R}$, $x > 0$.

- (d) (i) Write down an expression for $(f \circ g)(x)$.
- (ii) Solve the inequality $(f \circ g)(x) \leq 40$. [3]

— IB 2225 – 7109 Maths AA · May 2025 Paper 1 TZ1 SL, Question 8

18.

16 marks

[Maximum mark: 16]

Consider the function $f(x) = \frac{4x+2}{x-2}$, $x \neq 2$.

- (a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]
- (b) Write down the range of f . [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p , where $p \in \mathbb{Q}$.

- (c) Show that $p = \frac{9}{2}$. [1]
- (d) Find the value of b and the value of c . [3]
- (e) Find the y -coordinate of the vertex of the graph of $y = g(x)$. [2]
- (f) Find the product of the solutions of the equation $f(x) = g(x)$. [4]

— IB 2224 – 7101 Maths AA · May 2024 Paper 1 TZ1 HL, Question 10

19.

[Maximum mark: 16]

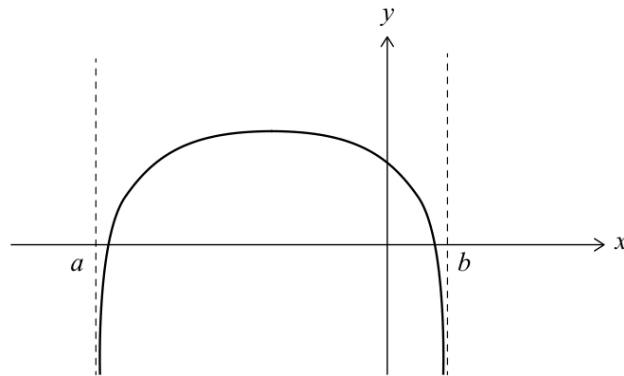
16 marks

(a) (i) Solve $5 - 4x - x^2 = 0$.

(ii) Hence, find the values of x such that $5 - 4x - x^2 > 0$.

[4]

Consider the function $f(x) = \log_k(5 - 4x - x^2)$, where $a < x < b$ and $k > 1$. Part of the graph of f is shown in the following diagram.



The graph of f has vertical asymptotes at $x = a$ and $x = b$.

(b) Write down the value of

(i) a ;

(ii) b .

[2]

(c) Find the exact values of x such that $f(x) = 0$.

[4]

The graph of f has a maximum value of 2.

(d) Find the value of k .

[6]

20. 18 marks

[Maximum mark: 18]

The function f is defined by $f(x) = 5(x + 1)(x + 3)$, where $x \in \mathbb{R}$.

- (a) Write $f(x)$ in the form $a(x - h)^2 + k$, where $a, h, k \in \mathbb{Z}$. [4]
- (b) Sketch the graph of $y = f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex. [4]
- (c) Solve the inequality $f(x) \leq 40$. [4]

The function g is defined by $g(x) = \ln x$, where $x \in \mathbb{R}$, $x > 0$.

- (d) (i) Write down an expression for $(f \circ g)(x)$.
- (ii) Solve the inequality $(f \circ g)(x) \leq 40$. [3]
- (e) Find the domain of $g \circ f$. [3]

— IB 2225 – 7106 Maths AA · May 2025 Paper 1 TZ1 HL, Question 10
