IB DIPLOMA · MATHS AA · SL

Rational and reciprocal functions

10 questions · 92 marks

Answer all questions. The marks available are shown against each part. Every question is taken unaltered from a past paper; the source is noted beneath it. Some questions are taken from higher level papers, where they examine the same standard level content.

1. 5 marks

[Maximum mark: 5]

The function f is defined by $f(x)=\frac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

(a) Write down the equation of

(i) the vertical asymptote of the graph of f;

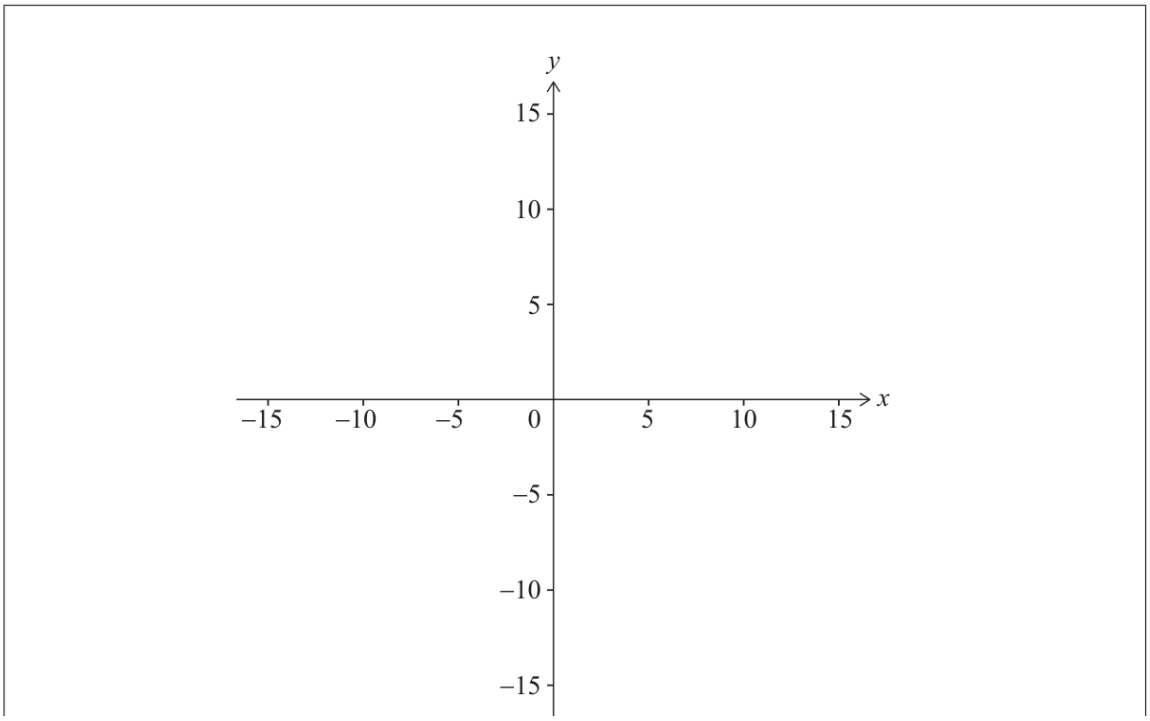
(ii) the horizontal asymptote of the graph of f. [2]

(b) Find the coordinates where the graph of f crosses

(i) the x-axis;

(ii) the y-axis. [2]

(c) Sketch the graph of f on the axes below. [1]



IB 8821 – 7104 Maths AA · November 2021 Paper 1 SL, Question 3

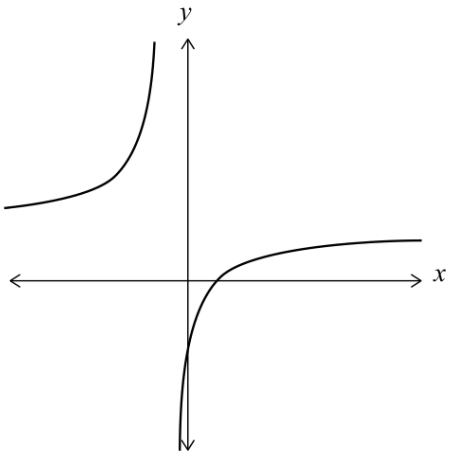
2.

5 marks

[Maximum mark: 5]

The function f is defined by $f(x)=\frac{3x-2}{2x+1}$ for $x \in \mathbb{R},\ x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of $y=f(x)$.



(a) Write down the value of $f(0)$. [1]

(b) Write down the equation of the horizontal asymptote. [1]

The function g is defined by $g(x)=-f(x)$ for $x \geq 0$.

(c) Find the range of g. [3]

IB 2225 – 7114 Maths AA · May 2025 Paper 1 TZ2 SL, Question 3

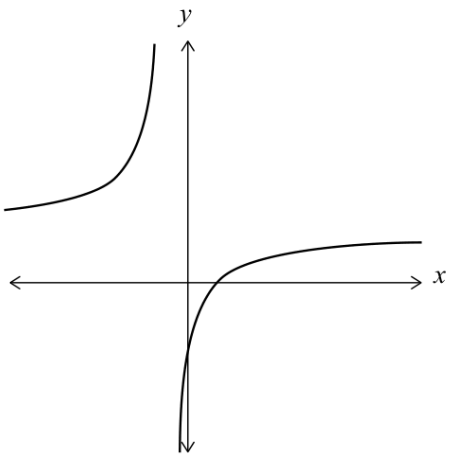
3.

5 marks

[Maximum mark: 5]

The function f is defined by $f(x)=\frac{3x-2}{2x+1}$ for $x \in \mathbb{R},\ x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of $y=f(x)$.



(a) Write down the value of $f(0)$. [1]

(b) Write down the equation of the horizontal asymptote. [1]

The function g is defined by $g(x)=-f(x)$ for $x \geq 0$.

(c) Find the range of g. [3]

IB 2225 – 7111 Maths AA · May 2025 Paper 1 TZ2 HL, Question 1

4. 7 marks

[Maximum mark: 7]

The function f is defined by $f(x)=\frac{7x+7}{2x-4}$ for $x \in \mathbb{R}$, $x \neq 2$.

(a) Find the zero of $f(x)$. [2]

(b) For the graph of $y=f(x)$, write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

IB 2223 – 7109 Maths AA · May 2023 Paper 1 TZ1 SL, Question 2

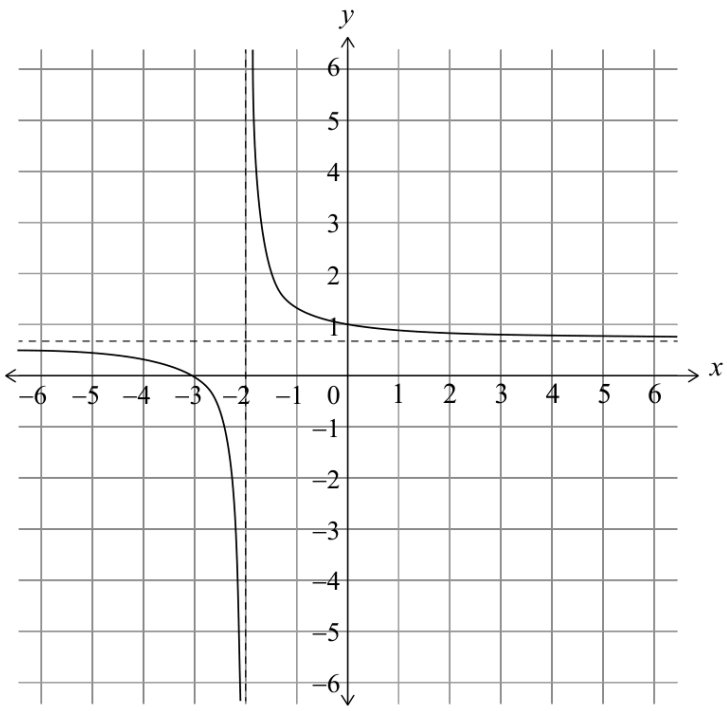
5.

8 marks

[Maximum mark: 8]

A function f is defined by $f(x)=\frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R}, x \neq -2$.

The graph $y=f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

Consider $g(x)=mx+1$, where $m \in \mathbb{R}, m \neq 0$.

(b) (i) Write down the number of solutions to $f(x)=g(x)$ for $m>0$.

(ii) Determine the value of m such that $f(x)=g(x)$ has only one solution for x.

(iii) Determine the range of values for m, where $f(x)=g(x)$ has two solutions for $x \geq 0$. [7]

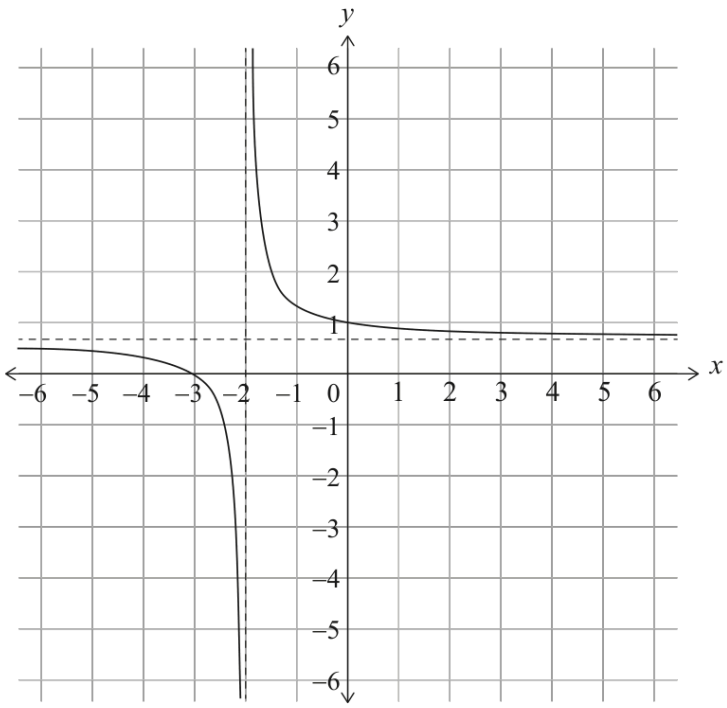
IB 2224 – 7109 Maths AA · May 2024 Paper 1 TZ2 SL, Question 6

6. 8 marks

[Maximum mark: 8]

A function f is defined by $f(x)=\frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R}, x \neq -2$.

The graph $y=f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

Consider $g(x)=mx+1$, where $m \in \mathbb{R}, m \neq 0$.

(b) (i) Write down the number of solutions to $f(x)=g(x)$ for $m>0$.

(ii) Determine the value of m such that $f(x)=g(x)$ has only one solution for x.

(iii) Determine the range of values for m, where $f(x)=g(x)$ has two solutions for $x \geq 0$. [7]

IB 2224 – 7106 Maths AA · May 2024 Paper 1 TZ2 HL, Question 5

7. 9 marks

[Maximum mark: 9]

The function f is defined by $f(x)=\frac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

(a) Write down the equation of

(i) the vertical asymptote of the graph of f;

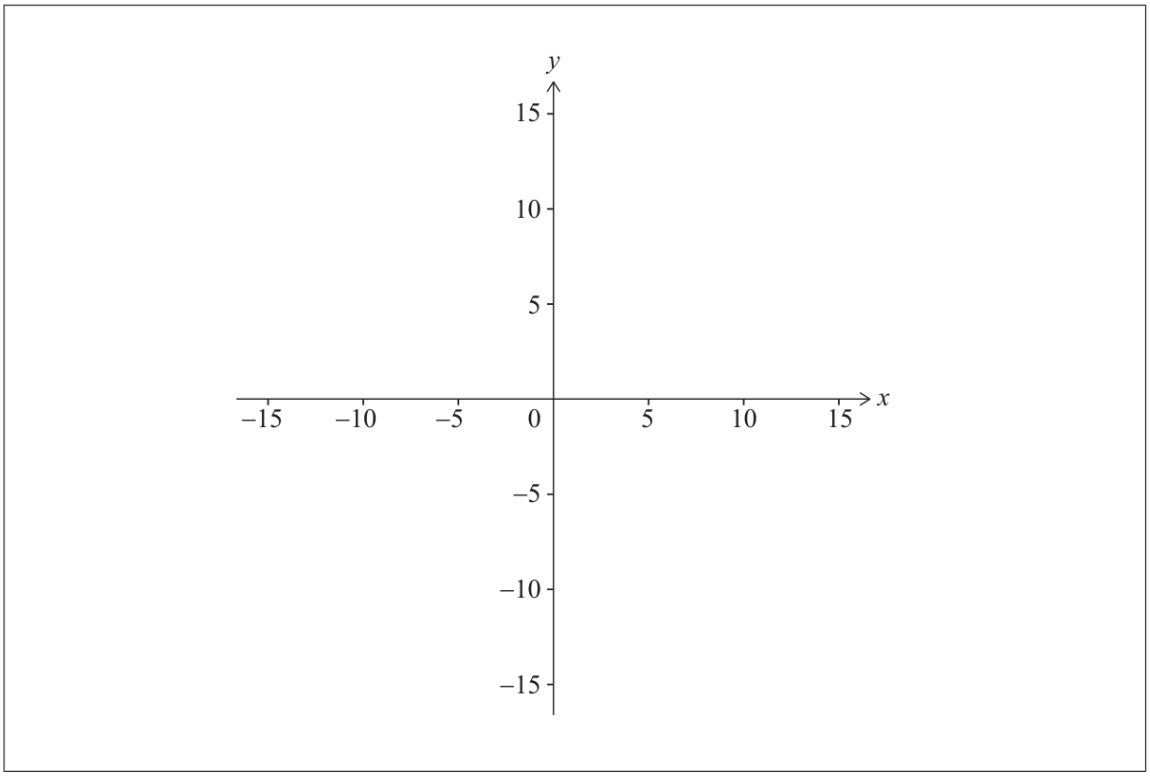
(ii) the horizontal asymptote of the graph of f. [2]

(b) Find the coordinates where the graph of f crosses

(i) the x-axis;

(ii) the y-axis. [2]

(c) Sketch the graph of f on the axes below. [1]



The function g is defined by $g(x)=\frac{ax+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$ and $a \in \mathbb{R}$.

(d) Given that $g(x)=g^{-1}(x)$, determine the value of a. [4]

IB 8821 – 7101 Maths AA · November 2021 Paper 1 HL, Question 2

8. 14 marks

[Maximum mark: 14]

Consider the function $f(x) = \frac{4x+2}{x-2}$, $x \neq 2$.

(a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]

(b) Write down the range of f. [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p, where $p \in \mathbb{Q}$.

(c) Show that $p = \frac{9}{2}$. [1]

(d) Find the value of b and the value of c. [3]

(e) Find the y-coordinate of the vertex of the graph of $y = g(x)$. [2]

(f) Find the number of solutions of the equation $f(x) = g(x)$. [2]

— IB 2224 – 7104 Maths AA · May 2024 Paper 1 TZ1 SL, Question 8

9. 15 marks

[Maximum mark: 15]

Consider the function $f(x) = \frac{2-2x}{x+2}$, where $x \in \mathbb{R}$, $x \neq -2$.

(a) Show that $f^{-1}(x) = f(x)$. [3]

The point $\mathrm{P}\left(k, \frac{2-2k}{k+2}\right)$ is the point on the graph of $y = f(x)$ that is closest to the origin $(0, 0)$.

(b) (i) Find the value of k.

(ii) Hence, write down the coordinates of P. [4]

Consider the function $g(x) = \frac{2-3x}{cx+d}$, where $x \in \mathbb{R}$, $x \neq -\frac{d}{c}$, and c, $d \neq 0$.

The graph of $y = g(x)$ has a vertical asymptote and a horizontal asymptote.

(c) In terms of c and d, write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

It is given that $g^{-1}(x) = g(x)$.

(d) Find the value of d. [2]

Consider the case where $c = 1$.

(e) Sketch the graph of $y = \frac{1}{g(x)}$, showing the values of any intercepts with the axes and including any asymptotes, labelled with their equations. [4]

— IB 2225 – 7110 Maths AA · May 2025 Paper 2 TZ1 SL, Question 9

10. 16 marks

[Maximum mark: 16]

Consider the function $f(x) = \frac{4x+2}{x-2}$, $x \neq 2$.

(a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]

(b) Write down the range of f. [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p, where $p \in \mathbb{Q}$.

(c) Show that $p = \frac{9}{2}$. [1]

(d) Find the value of b and the value of c. [3]

(e) Find the y-coordinate of the vertex of the graph of $y = g(x)$. [2]

(f) Find the product of the solutions of the equation $f(x) = g(x)$. [4]

IB 2224 – 7101 Maths AA · May 2024 Paper 1 TZ1 HL, Question 10