CuratedPrep

IB DIPLOMA · MATHS AA · SL

Arithmetic sequences and series

16 questions · 159 marks

Answer all questions. The marks available are shown against each part. Every question is taken unaltered from a past paper; the source is noted beneath it. Some questions are taken from higher level papers, where they examine the same standard level content.

1.

4 marks

[Maximum mark: 4]

The second term of an arithmetic sequence is 10 and the fourth term is 22.

(a) Find the value of the common difference. [2]

(b) Find an expression for u_n, the nth term. [2]

— IB 2224 – 7104 Maths AA · May 2024 Paper 1 TZ1 SL, Question 1

2.

5 marks

[Maximum mark: 5]

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d.

— IB 2221 – 7109 Maths AA · May 2021 Paper 1 TZ1 SL, Question 3

3. 5 marks

[Maximum mark: 5]

An arithmetic sequence has first term 60 and common difference -2.5.

(a) Given that the kth term of the sequence is zero, find the value of k. [2]

Let S_n denote the sum of the first n terms of the sequence.

(b) Find the maximum value of S_n. [3]

— IB 2221 – 7115 Maths AA · May 2021 Paper 2 TZ2 SL, Question 3

4. 5 marks

[Maximum mark: 5]

An arithmetic sequence has first term 60 and common difference -2.5.

(a) Given that the kth term of the sequence is zero, find the value of k. [2]

Let S_n denote the sum of the first n terms of the sequence.

(b) Find the maximum value of S_n. [3]

— IB 2221 – 7112 Maths AA · May 2021 Paper 2 TZ2 HL, Question 2

5. 5 marks

[Maximum mark: 5]

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d.

— IB 2221 – 7106 Maths AA · May 2021 Paper 1 TZ1 HL, Question 2

6. 6 marks

[Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 14$ and $S_{25} = 200$.

Find the value of k such that $u_k = 0$.

— IB 8824 – 7104 Maths AA · November 2024 Paper 1 TZ1 SL, Question 6

7. 6 marks

[Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 16$ and $S_{25} = 100$.

Find the value of k such that $u_k = 0$.

— IB 8824 – 7109 Maths AA · November 2024 Paper 1 TZ2 SL, Question 6

8. 6 marks

[Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 16$ and $S_{25} = 100$.

Find the value of k such that $u_k = 0$.

— IB 8824 – 9700 Maths AA · November 2024 Paper 1 HL, Question 5

9. 7 marks

[Maximum mark: 7]

A geometric sequence has first term 80 and fourth term 0.74088.

(a) Find the second term. [3]

The first two terms of this geometric sequence are also the first term and eleventh term respectively, of an arithmetic sequence.

Let S_n denote the sum of the first n terms of the arithmetic sequence.

(b) Find the greatest value of S_n, giving your answer to two decimal places. [4]

— IB 2225 – 7112 Maths AA · May 2025 Paper 2 TZ2 HL, Question 7

10. 14 marks

[Maximum mark: 14]

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a) (i) Find the sum of the first five terms.

(ii) Given that $S_6 = 60$, find u_6. [4]

(b) Find u_1. [2]

(c) Hence or otherwise, write an expression for u_n in terms of n. [3]

Consider a geometric sequence, v_n, where $v_2 = u_1$ and $v_4 = u_6$.

(d) Find the possible values of the common ratio, r. [3]

(e) Given that $v_{99} < 0$, find v_5. [2]

— IB 2223 – 7109 Maths AA · May 2023 Paper 1 TZ1 SL, Question 8

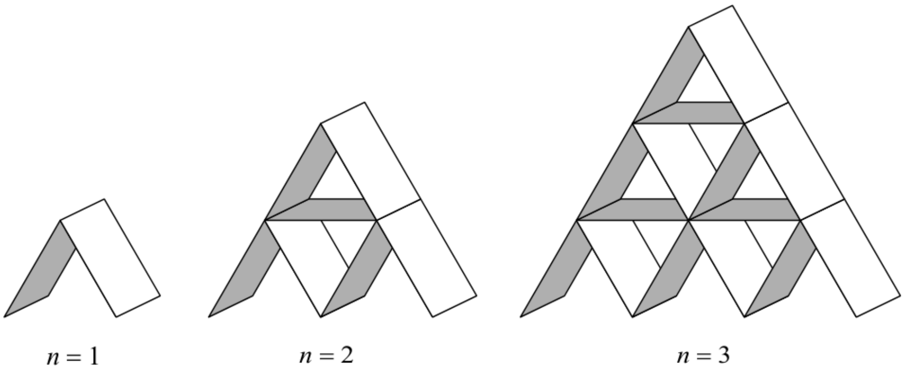
11. 16 marks

[Maximum mark: 16]

Rectangular playing cards are stacked in the shape of a pyramid with n rows, where $n \geq 1$.

Some cards are placed horizontally and some cards are stacked at an angle of $60°$ to the horizontal.

The following diagrams represent pyramid stacks for $n = 1$, $n = 2$ and $n = 3$.



Let t_n represent the number of cards used to create a pyramid stack with n rows.

(a) Write down t_3. [1]

(b) Find t_4. [2]

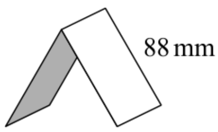
(c) Show that $t_n = \dfrac{n(3n+1)}{2}$. [3]

There are 52 cards in a full pack of playing cards.

(d) A complete pyramid stack is created using playing cards taken from 14 full packs. Find the maximum number of rows in this stack. [3]

(e) A complete pyramid stack is created using playing cards taken from full packs with no cards left over. Find the minimum number of rows in this stack. [2]

The long edge of each playing card measures $88\,\text{mm}$ as illustrated in the following diagram.



(f) Find the minimum number of cards needed to create a complete pyramid stack with a vertical height of more than 2 metres. The thickness of the cards may be ignored. [5]

— IB 2225 – 7110 Maths AA · May 2025 Paper 2 TZ1 SL, Question 8

12. [Maximum mark: 16] 16 marks

Consider the arithmetic sequence a, p, $q\ldots$, where a, p, $q \neq 0$.

(a) Show that $2p - q = a$. [2]

Consider the geometric sequence a, s, $t\ldots$, where a, s, $t \neq 0$.

(b) Show that $s^2 = at$. [2]

The first term of both sequences is a.

It is given that $q = t = 1$.

(c) Show that $p > \frac{1}{2}$. [2]

Consider the case where $a = 9$, $s > 0$ and $q = t = 1$.

(d) Write down the first four terms of the

(i) arithmetic sequence;

(ii) geometric sequence. [4]

The arithmetic and the geometric sequence are used to form a new arithmetic sequence u_n.

The first three terms of u_n are $u_1 = 9 + \ln 9$, $u_2 = 5 + \ln 3$, and $u_3 = 1 + \ln 1$.

(e) (i) Find the common difference of the new sequence in terms of $\ln 3$.

(ii) Show that $\sum_{i=1}^{10} u_i = -90 - 25\ln 3$. [6]

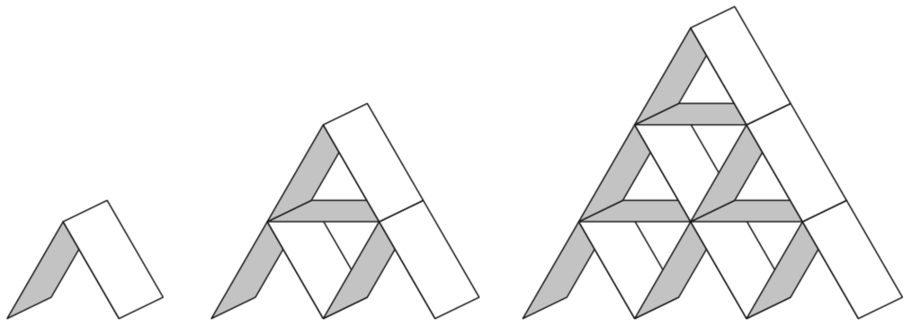
— IB 2224 – 7109 Maths AA · May 2024 Paper 1 TZ2 SL, Question 9

13. [Maximum mark: 16] 16 marks

Rectangular playing cards are stacked in the shape of a pyramid with n rows, where $n \geq 1$.

Some cards are placed horizontally and some cards are stacked at an angle of $60°$ to the horizontal.

The following diagrams represent pyramid stacks for $n = 1$, $n = 2$ and $n = 3$.



$n = 1$ $n = 2$ $n = 3$

Let t_n represent the number of cards used to create a pyramid stack with n rows.

(a) Write down t_3. [1]

(b) Find t_4. [2]

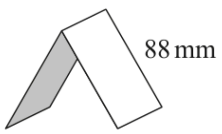
(c) Show that $t_n = \dfrac{n(3n+1)}{2}$. [3]

There are 52 cards in a full pack of playing cards.

(d) A complete pyramid stack is created using playing cards taken from 14 full packs. Find the maximum number of rows in this stack. [3]

(e) A complete pyramid stack is created using playing cards taken from full packs with no cards left over. Find the minimum number of rows in this stack. [2]

The long edge of each playing card measures $88\,\text{mm}$ as illustrated in the following diagram.



(f) Find the minimum number of cards needed to create a complete pyramid stack with a vertical height of more than 2 metres. The thickness of the cards may be ignored. [5]

— IB 2225 – 7107 Maths AA · May 2025 Paper 2 TZ1 HL, Question 10

14. [Maximum mark: 16] 16 marks

Consider the arithmetic sequence a, p, q ..., where $a, p, q \neq 0$.

(a) Show that $2p - q = a$. [2]

Consider the geometric sequence a, s, t ..., where $a, s, t \neq 0$.

(b) Show that $s^2 = at$. [2]

The first term of both sequences is a.

It is given that $q = t = 1$.

(c) Show that $p > \frac{1}{2}$. [2]

Consider the case where $a = 9$, $s > 0$ and $q = t = 1$.

(d) Write down the first four terms of the

(i) arithmetic sequence;

(ii) geometric sequence. [4]

The arithmetic and the geometric sequence are used to form a new arithmetic sequence u_n.

The first three terms of u_n are $u_1 = 9 + \ln 9$, $u_2 = 5 + \ln 3$, and $u_3 = 1 + \ln 1$.

(e) (i) Find the common difference of the new sequence in terms of $\ln 3$.

(ii) Show that $\sum_{i=1}^{10} u_i = -90 - 25\ln 3$. [6]

— IB 2224 – 7106 Maths AA · May 2024 Paper 1 TZ2 HL, Question 10

15. [Maximum mark: 16] 16 marks

Consider the sequence $\{u_n\}$, with nth term given by u_n. The first three terms are

$$u_1 = k - 5,\ u_2 = 3 - 2k \text{ and } u_3 = 5k + 3, \text{ where } k \in \mathbb{R}.$$

(a) Consider the case when $\{u_n\}$ is arithmetic.

(i) Find the value of k.

(ii) Hence, or otherwise, find u_3. [5]

(b) Consider the case where $k = 12$.

(i) Show that the first three terms of $\{u_n\}$ form a geometric sequence.

(ii) Given that $\{u_n\}$ is geometric, state a reason why the sum of an infinite number of terms of this sequence does not exist. [4]

(c) The sequence, $\{u_n\}$, is geometric for a second value of k.

(i) Show that $k^2 - 10k - 24 = 0$.

(ii) Find the first three terms of $\{u_n\}$ for this second value of k.

(iii) Hence, write down the value of S_{2m}, the sum of the first $2m$ terms, for this second value of k. [7]

— IB 2225 – 7119 Maths AA · May 2025 Paper 1 TZ3 SL, Question 8

16. 16 marks

[Maximum mark: 16]

Consider the sequence $\{u_n\}$, with $n\text{th}$ term given by u_n. The first three terms are

$$u_1 = k - 5,\ u_2 = 3 - 2k \text{ and } u_3 = 5k + 3, \text{ where } k \in \mathbb{R}.$$

(a) Consider the case when $\{u_n\}$ is arithmetic.

(i) Find the value of k.

(ii) Hence, or otherwise, find u_3. [5]

(b) Consider the case where $k = 12$.

(i) Show that the first three terms of $\{u_n\}$ form a geometric sequence.

(ii) Given that $\{u_n\}$ is geometric, state a reason why the sum of an infinite number of terms of this sequence does not exist. [4]

(c) The sequence, $\{u_n\}$, is geometric for a second value of k.

(i) Show that $k^2 - 10k - 24 = 0$.

(ii) Find the first three terms of $\{u_n\}$ for this second value of k.

(iii) Hence, write down the value of S_{2m}, the sum of the first $2m$ terms, for this second value of k. [7]

— IB 2225 – 7116 Maths AA · May 2025 Paper 1 TZ3 HL, Question 10